13 Probability

Fastrack Revision

- **Probability of an Event:** In a random experiment, let S be the random space and let $E \subseteq S$. Then E is an event. The probability of occurrence of E is defined as:

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

- **Conditional Probability**

Suppose E and F are two events associated with the same random experiment, then the probability of occurrence of E under the condition that F has already occurred and $P(F) \neq 0$, is called conditional probability, denoted by $P(E/F)$.

- **Properties of Conditional Probability**
  - $P(E/F) = \frac{P(E \cap F)}{P(F)}$, where, $P(F) \neq 0$
  - $P(F/E) = \frac{P(E \cap F)}{P(E)}$, where, $P(E) \neq 0$
  - $P(E'/F) = 1 - P(E/F)$
  - $P[(E \cup F)/G] = P(E/G) + P(F/G) - P[(E \cap F)/G]$
  - $P(S/F) = P(F/F) = 1$
  - Two events E and F associated to a random experiment are **independent**, if the probability of occurrence or non-occurrence of E is not affected by the occurrence or non-occurrence of F.
  - Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) \cdot P(F)$.
  - If E and F are independent events, then $P(E \cap F) = P(E) \cdot P(F) \Rightarrow P(E/F) = P(E), P(F) \neq 0$ and $P(F/E) = P(F), P(E) \neq 0$

- **Multiplication Rule of Probability**
  - $P(E \cap F) = P(E) \cdot P(F/E)$, provided $P(E) \neq 0$
  - $P(E \cap F) = P(F) \cdot P(E/F)$, provided $P(F) \neq 0$
  - **Theorem of Total Probability:** Let $\{E_1, E_2, E_3, ..., E_n\}$ be a partition of sample spaces and suppose that each of $E_1, E_2, E_3, ..., E_n$ has non-zero probability. Let A be any event associated with S, then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + ... + P(E_n) \cdot P(A/E_n) = \sum_{j=1}^{n} P(E_j) \cdot P(A/E_j)$$

- **Bayes' Theorem:** If $E_1, E_2, E_3, ..., E_n$ are events which constitute a partition of sample space S, i.e., $E_1, E_2, E_3, ..., E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup E_3 \cup ... \cup E_n = S$ and A be any event with non-zero probability, then

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$ for any $i = 1, 2, 3, ..., n$.

- A **random variable** is a real valued function, whose domain is the sample space of a random experiment.
- **Probability Distribution of a Random Variable**

A description giving the values of a random variable along with the corresponding probabilities is called the probability distribution of the random variable.

If a random variable X takes the values $x_1, x_2, ..., x_n$ with respective probabilities $p_1, p_2, ..., p_n$ then the probability distribution of X is given by

X	x_1	x_2	x_3	......	x_n
$P(X)$	p_1	p_2	p_3	......	p_n

where, $p_i > 0$ and $\sum_{i=1}^{n} P_i = 1$.

- Let X be a random variable whose possible values are $x_1, x_2, x_3, ..., x_n$ with probabilities $p_1, p_2, p_3, ..., p_n$ respectively. The **mean** μ of X is given by $\sum_{i=1}^{n} p_i x_i$. It is also called the **expectation of X,** denoted by $E(X)$.

Knowledge BOOSTER

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. $P(A \cup B) + P(\overline{A \cup B}) = 1$
3. $P(A' \cap B') = 1 - P(A \cup B)$

Practice Exercise

Multiple Choice Questions

Q 1. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $P(B/A)$ is equal to: *(NCERT EXEMPLAR; CBSE 2023)*

a. $\frac{1}{10}$ b. $\frac{1}{8}$ c. $\frac{7}{8}$ d. $\frac{17}{20}$

Q 2. For two events A and B, if $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B)$ is: *(CBSE 2023)*

a. 0.24 b. 0.3 c. 0.48 d. 0.96

Q 3. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A'/B') \cdot P(B'/A')$ is equal to: *(NCERT EXEMPLAR)*

a. $\frac{5}{6}$ b. $\frac{5}{7}$ c. $\frac{25}{42}$ d. 1

Q 4. The events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then $P(E/F) - P(F/E)$ equals: *(CBSE 2023; NCERT EXEMPLAR)*

a. $\frac{1}{7}$ b. $\frac{2}{7}$ c. $\frac{3}{35}$ d. $\frac{1}{70}$

Q 5. If A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(B'/A)$ is: *(CBSE 2023)*

a. $\frac{1}{4}$ b. $\frac{1}{8}$ c. $\frac{3}{4}$ d. 1

Q 6. Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$ and $P(A' \cap B')$ is: *(CBSE SQP 2022-23)*

a. 0.9 b. 0.18
c. 0.28 d. 0.1

Q 7. If A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$, then $P(B' \cap A)$ equals to: *(NCERT EXEMPLAR)*

a. $\frac{2}{3}$ b. $\frac{1}{2}$ c. $\frac{3}{10}$ d. $\frac{1}{5}$

Q 8. If A and B are two events such that $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A)$ equals to: *(NCERT EXEMPLAR)*

a. $\frac{3}{10}$ b. $\frac{1}{5}$ c. $\frac{1}{2}$ d. $\frac{3}{5}$

Q 9. If $P(B) = \frac{3}{5}$, $P(A/B) = \frac{1}{2}$ and $P(A \cup B) = \frac{4}{5}$, then $P(A \cup B)' + P(A' \cup B)$ is equal to: *(NCERT EXEMPLAR)*

a. $\frac{1}{5}$ b. $\frac{4}{5}$ c. $\frac{1}{2}$ d. 1

Q 10. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then $P(A'/B)$ is equal to: *(NCERT EXEMPLAR)*

a. $\frac{6}{13}$ b. $\frac{4}{13}$ c. $\frac{4}{9}$ d. $\frac{5}{9}$

Q 11. If A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then $P(A/B) \cdot P(A'/B)$ is equal to: *(NCERT EXEMPLAR)*

a. $\frac{2}{5}$ b. $\frac{3}{8}$ c. $\frac{3}{20}$ d. $\frac{6}{25}$

Q 12. If E and F are events such that $0 < P(F) < 1$, then:

a. $P(E|F) + P(\bar{E}|F) = 1$ b. $P(E|F) + P(E|\bar{F}) = 1$
c. $P(\bar{E}|F) + P(E|\bar{F}) = 1$ d. $P(E|\bar{F}) + P(\bar{E}|\bar{F}) = 0$

Q 13. Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

a. $\frac{1}{42}$ b. $\frac{2}{21}$ c. $\frac{1}{18}$ d. $\frac{1}{21}$

Q 14. If six cards are selected at random (without replacement) from a standard deck of 52 cards, what is the probability that there will be no pairs? (two cards of same denomination)

a. 0.28 b. 0.562
c. 0.345 d. 0.832

Q 15. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is: *(CBSE 2023)*

a. $\frac{27}{32}$ b. $\frac{5}{32}$ c. $\frac{31}{32}$ d. $\frac{1}{32}$

Q 16. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement, then the probability that both drawn balls are black, is:

a. $\frac{2}{7}$ b. $\frac{1}{7}$ c. $\frac{5}{7}$ d. $\frac{3}{7}$

Q 17. For any two events A and B:

a. $P(A/B) \geq \frac{P(A) + P(B) + 1}{P(B)}$

b. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ does not hold

c. $P(A \cup B) = 1 - P(\bar{A}) \cdot P(\bar{B})$, if A and B are independent

d. $P(A \cup B) = 1 - P(\bar{A}) \cdot P(\bar{B})$, if A and B are dependent

Q 18. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.

a. $\frac{1}{4}$ b. $\frac{3}{5}$ c. $\frac{2}{3}$ d. $\frac{2}{5}$

Q 19. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$ equals to: *(NCERT EXEMPLAR)*

a. $\frac{4}{15}$ b. $\frac{8}{45}$ c. $\frac{1}{3}$ d. $\frac{2}{9}$

Q 20. The probability that A speaks the truth is $\frac{4}{5}$ and that of B speaking the truth is $\frac{3}{4}$. The probability that they contradict each other in stating the same fact is: *(CBSE 2023)*

a. $\frac{7}{20}$ b. $\frac{1}{5}$ c. $\frac{3}{20}$ d. $\frac{4}{5}$

Q 21. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is: *(CBSE SQP 2023-24)*

a. $\frac{1}{4}$ b. $\frac{1}{3}$
c. $\frac{1}{2}$ d. $\frac{3}{4}$

Q 22. A university has to select an examiner from a list of 50 persons, 20 of them are women and 30 men, 10 of them knowing Hindi and 40 not, 15 of them being teachers and the remaining 35 not. What is the probability of the university selecting a Hindi knowing woman teacher?

a. $\frac{1}{125}$ b. $\frac{2}{125}$ c. $\frac{4}{125}$ d. $\frac{3}{125}$

Q 23. One half per cent of the population has a particular disease. A test is developed for the disease. The test gives a false positive 3% of the time and a false negative 2% of the time. What is the probability that Amit (a random person) tests positive?

a. 0.476 b. 0.035 c. 0.523 d. 0.232

Q 24. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is: *(NCERT EXEMPLAR)*

a. $\frac{45}{196}$ b. $\frac{135}{392}$ c. $\frac{15}{56}$ d. $\frac{15}{29}$

Q 25. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has atleast one girl is: *(NCERT EXEMPLAR)*

a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{2}{3}$ d. $\frac{4}{7}$

Q 26. If a die is thrown and a card is selected at random from a deck of 52 playing cards, then the probability of getting an even number on the die and a spade card is: *(NCERT EXEMPLAR)*

a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{8}$ d. $\frac{3}{4}$

Q 27. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is: *(NCERT EXEMPLAR)*

a. $\frac{3}{28}$ b. $\frac{2}{21}$ c. $\frac{1}{28}$ d. $\frac{167}{168}$

Q 28. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is: *(NCERT EXEMPLAR)*

a. $\frac{1}{18}$ b. $\frac{5}{18}$ c. $\frac{1}{5}$ d. $\frac{2}{5}$

Q 29. *A* and *B* are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is, $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is: *(NCERT EXEMPLAR)*

a. $\frac{1}{12}$ b. $\frac{1}{40}$ c. $\frac{13}{120}$ d. $\frac{10}{13}$

Q 30. The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
$P(X)$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

The value of k is: *(NCERT EXEMPLAR)*

a. 8 b. 16
c. 32 d. 48

Q 31. For the following probability distribution:

X	−4	−3	−2	−1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to:

a. 0 b. −1 c. −2 d. −1.8

Q 32. For the following probability distribution:

X	1	2	3	4
$P(X)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2)$ is equal to: *(NCERT EXEMPLAR)*

a. 3 b. 5 c. 7 d. 10

Q 33. Let X denotes the number of hours you study on a Sunday. Also it is known that

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

What is the probability that you study atleast two hours?

a. 0.55 b. 0.15 c. 0.6 d. 0.3

Q 34. In a college, 30% students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in Physics, if she has failed in Mathematics is: *(NCERT EXEMPLAR)*

a. $\frac{1}{10}$ b. $\frac{2}{5}$ c. $\frac{9}{20}$ d. $\frac{1}{3}$

Assertion & Reason Type Questions

Directions (Q. Nos. 35-42): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 35. Assertion (A): Two coins are tossed simultaneously. The probability of a getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason (R): Let E and F be two events with a random experiment, then $P(F / E) = \frac{P(E \cap F)}{P(E)}$.

(CBSE 2023)

Q 36. Assertion (A): Consider the experiment of drawing a card from a deck of 52 playing cards, in which the elementary events are assumed to be equally likely. If E and F denote the events the card drawn is a spade and the card drawn is an ace respectively, then $P(E/F) = \frac{1}{4}$ and $P(F/E) = \frac{1}{13}$.

Reason (R): E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other. Such events are called independent events.

Q 37. Assertion (A): Let E and F be events associated with the sample space S of an experiment. Then, we have $P(S/F) = P(F/F) = 1$.

Reason (R): If A and B are any two events associated with the sample space S and F is an event associated with S such that $P(F) \neq 0$, then $P((A \cup B)/F) = P(A/F) + P(B/F) - P((A \cap B)/F)$.

Q 38. Let A and B be two events associated with an experiment such that $P(A \cap B) = P(A)\,P(B)$.

Assertion (A): $P(A/B) = P(A)$ and $P(B/A) = P(B)$

Reason (R): $P(A \cup B) = P(A) + P(B)$

Q 39. Let $H_1, H_2, \ldots\ldots, H_n$ be mutually exclusive and exhaustive events with $P(H_i) > 0$, $i = 1, 2, \ldots, n$.

Let E be any other event with $0 < P(E) < 1$

Assertion (A): $P(H_i / E) > P(E / H_i) \times P(H_i)$ for $i = 1, 2, \ldots\ldots, n$

Reason (R): $\sum_{i=1}^{n} P(H_i) = 1$

Q 40. Assertion (A): An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. Then, the probability that the second ball is red is $\frac{1}{2}$.

Reason (R): A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Then, the probability that the ball is drawn from the first bag is $\frac{2}{3}$.

Q 41. Assertion (A): The mean of a random variable X is also called the expectation of X, denoted by $E(X)$.

Reason (R): The mean or expectation of a random variable X is not sum of the probabilities of all possible values of X by their respective probabilities.

Q 42. Assertion (A): The mean number of heads in three tosses of a fair coin is 1.5.

Reason (R): Two dice are thrown simultaneously. If X denotes the number of sixes, the expectation of X is $\frac{1}{3}$.

Answers

1. (c)	2. (d)	3. (c)	4. (d)	5. (c)	6. (c)	7. (d)	8. (c)	9. (d)	10. (d)
11. (d)	12. (a)	13. (d)	14. (c)	15. (c)	16. (d)	17. (c)	18. (d)	19. (d)	20. (a)
21. (d)	22. (d)	23. (b)	24. (c)	25. (d)	26. (c)	27. (a)	28. (c)	29. (d)	30. (c)
31. (d)	32. (d)	33. (c)	34. (b)	35. (a)	36. (a)	37. (b)	38. (c)	39. (d)	40. (b)
41. (c)	42. (b)								

Case Study Based Questions

Case Study 1

The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive.

Based on the given information, solve the following questions:

Q 1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?

a. 0.001 b. 0.1 c. 0.8 d. 0.9

Q 2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

a. 0.01 b. 0.99 c. 0.1 d. 0.001

Q 3. What is the probability that the 'person is actually not having COVID'?

a. 0.998 b. 0.999 c. 0.001 d. 0.111

Q 4. What is the probability that the 'person is actually having COVID' given that 'he is tested as COVID positive'?

a. 0.83 b. 0.0803 c. 0.083 d. 0.089

Q 5. What is the probability that the 'person selected will be diagnosed as COVID positive'?

a. 0.1089 b. 0.01089 c. 0.0189 d. 0.189

Solutions

Consider the events

E_1 = People having COVID,

E_2 = People free of COVID

A = Person to be tested as COVID positive

1. Probability of the 'person to be tested as COVID positive' given that he is actually having COVID

$$= P\left(\frac{A}{E_1}\right) = 90\% = \frac{90}{100} = 0.9$$

So, option (d) is correct.

2. Probability of the 'person to be tested as COVID positive' given that he is actually not having COVID

$$= P\left(\frac{A}{E_2}\right) = 1\% = \frac{1}{100} = 0.01$$

So, option (a) is correct.

3. Probability that the person is actually not having COVID $= P(E_2) = 99.9\% = \frac{999}{1000} = 0.999$

So, option (b) is correct.

4. Probability that the person is actually having COVID given that he is tested as COVID positive

$$= P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{0.1\% \times 90\%}{0.1\% \times 90\% + 99.9\% \times 1\%}$$

$$= \frac{0.1 \times 90}{0.1 \times 90 + 99.9 \times 1}$$

$$= \frac{9}{9 + 99.9} = \frac{9}{108.9} = 0.0826$$

$\therefore$ Required probability = 0.083

So, option (c) is correct.

5. Probability that the person selected will be diagonosed as COVID positive

$$P(A) = P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)$$

$$= 0.1\% \times 90\% + 99.9\% \times 1\%$$

$$= \frac{90}{100000} + \frac{999}{100000} = \frac{1089}{100000} = 0.01089$$

So, option (b) is correct.

Case Study 2

An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 per cent of the population is accident prone.

Based on the above information, solve the following questions: (CBSE SQP 2022 Term-2)

Q 1. What is the probability that a new policyholder will have an accident within a year of purchasing a policy?

Q 2. Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone?

Solutions

1. Let E_1 = The policy holder is accident prone.

E_2 = The policy holder is not accident prone.

E = The new policy holder has an accident within a year of purchasing a policy.

$$\therefore \quad P(E) = P(E_1) \times P(E/E_1) + P(E_2) \times P(E/E_2)$$

$$= \frac{6}{10} \times \frac{20}{100} + \frac{2}{10} \times \frac{80}{100} = \frac{280}{1000} = \frac{7}{25}$$

2. By Bayes' theorem,

$$P(E_1/E) = \frac{P(E_1) \times P(E/E_1)}{P(E)}$$

$$= \frac{\frac{6}{10} \times \frac{20}{100}}{\frac{280}{1000}} = \frac{3}{7}$$

Case Study 3

A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information, solve the following questions: (CBSE SQP 2022 Term-2)

Q 1. Calculate the probability that a randomly chosen seed will germinate.

Q 2. Calculate the probaility that the seed is of type A_2, given that a randomly chosen seed germinates.

Solutions

1. We have, $A_1 : A_2 : A_3 = 4 : 4 : 2$

$P(A_1) = \frac{4}{10}, P(A_2) = \frac{4}{10}$ and $P(A_3) = \frac{2}{10}$

where A_1, A_2 and A_3 denote the three types of flower seeds.

Let E be the event that a seed germinates.

$\therefore \quad P(E/A_1) = \frac{45}{100}, P(E/A_2) = \frac{60}{100}$

and $\quad P(E/A_3) = \frac{35}{100}$

$\therefore \quad P(E) = P(A_1)\cdot P(A/E_1)$

$P(A_2)\cdot P(A/E_2) + P(A_3)\cdot P(A/E_3)$

$= \frac{4}{10}\times\frac{45}{100} + \frac{4}{10}\times\frac{60}{100} + \frac{2}{10}\times\frac{35}{100}$

$= \frac{490}{1000} = 0.49$

2. $$P\left(\frac{A_2}{E}\right) = \frac{P(A_2)\cdot P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2) P(E/A_2) + P(A_3)P(E/A_3)}$$

[By Baye's theorem]

$$= \frac{\frac{4}{10}\times\frac{60}{100}}{\frac{4}{10}\times\frac{45}{100} + \frac{4}{10}\times\frac{60}{100} + \frac{2}{10}\times\frac{35}{100}}$$

$$= \frac{240/1000}{490/1000}$$

$$= \frac{240}{490} = \frac{24}{49}.$$

Case Study 4

There are two antiaircraft guns, name as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

Based on the above information, solve the following questions: (CBSE SQP 2022-23)

Q 1. What is the probability that the shell fired from exactly one of them hit the plane?

Q 2. If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

Solutions

1. P (Shell fired from exactly one of them hits the plane)
$= P$ [(Shell from A hits the plane and shell from B does not hit the plane) or (shell from A does not hit the plane and shell from B hits the plane)]
$= 0.3 \times 0.8 + 0.7 \times 0.2 = 0.24 + 0.14 = 0.38.$

2. P (Shell fired from B hit the plane/Exactly one of them hit the plane)

$$= \frac{P(\text{Shell fired from } B \text{ hit the plane} \cap \text{Exactly one of them hit the plane})}{P(\text{Exactly one of them hit the plane})}$$

$$= \frac{P(\text{Shell from only } B \text{ hit the plane})}{P(\text{Exactly one of them hit the plane})}$$

$$= \frac{0.14}{0.38} = \frac{7}{19}.$$

$[\because P(\overline{A} \cap B) = P(\overline{A}) \times P(B) = (1-0.3)\times 0.2 = 0.14]$

Case Study 5

A building contractor undertakes a job to construct 4 flats on a plot alongwith parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let E_1: represent the event when many workers were not present for the job

E_2: represent the event when all workers were present and

E : represent completing the construction work on time. (CBSE 2023)

Based on the above information, solve the following questions:

Q 1. What is the probability that all the workers are present for the job?

Q 2. What is the probability that construction will be completed on time?

Q 3. What is the probability that many workers are not present given that the construction work is completed on time?

Or

What is the probability that all workers were present given that the construction job was completed on time?

Solutions

1. Given $P(E_1) = 0.65, P\left(\frac{E}{E_1}\right) = 0.35, P\left(\frac{E}{E_2}\right) = 0.80$

$\therefore \quad P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

2. $P(E) = P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right)$

$= 0.65 \times 0.35 + 0.35 \times 0.80$

$= 0.2275 + 0.28 = 0.5075$

3. $P\left(\frac{E}{E_1}\right) = 0.35$

Or

$P\left(\frac{E}{E_2}\right) = 0.80$

Case Study 6

In an office three employees James, Sophia and Oliver process incoming copies of a certain form. James processes 50% of the forms, Sophia processes 20% and Oliver the remaining 30% of the forms. James has an error rate of 0.06, Sophia has an error rate of 0.04 and Oliver has an error rate of 0.03.

Based on the above information, solve the following questions: (CBSE SQP 2023-24)

Q 1. Find the probability that Sophia processed the form and committed an error.

Q 2. Find the total probability of committing an error in processing the form.

Q 3. The manager of the company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is not processed by James.

Or

Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that James, Sophia and Oliver processed the form. Find the value of $\sum_{i=1}^{3} P(E_i / E)$.

Solutions

Given $P(J) = 50\% = \frac{50}{100} = \frac{1}{2}$

$P(S) = 20\% = \frac{20}{100} = \frac{1}{5}$

$P(O) = 30\% = \frac{30}{100} = \frac{3}{10}$

$P\left(\frac{E}{J}\right) = 0.06, P\left(\frac{E}{S}\right) = 0.04 \text{ and } P\left(\frac{E}{O}\right) = 0.03$

1. Probability that Sophia processed the form and committed an error $= P(S)P\left(\frac{E}{S}\right)$

$= \frac{1}{5} \times 0.04 = 0.008$

2. The total probability of committing an error in processing the form

$= P(J)P\left(\frac{E}{J}\right) + P(S)P\left(\frac{E}{S}\right) + P(O)P\left(\frac{E}{O}\right)$

$= \frac{1}{2} \times 0.06 + \frac{1}{5} \times 0.04 + \frac{3}{10} \times 0.03$

$= 0.03 + 0.008 + 0.009$

$= 0.047$

3. Probability that an error is committed by James in processing,

$$P\left(\frac{J}{E}\right) = \frac{P(J) \times P\left(\frac{E}{J}\right)}{P(J)P\left(\frac{E}{J}\right) + P(S)P\left(\frac{E}{S}\right) + P(O)P\left(\frac{E}{O}\right)}$$

$$= \frac{\frac{1}{2} \times 0.06}{0.047} = \frac{3 \times 10}{47} = \frac{30}{47}$$

$\therefore$ Required probability that error was not processed by James $= 1 - P\left(\frac{J}{E}\right) = 1 - \frac{30}{47} = \frac{17}{47}$

Or

$$\sum_{i=1}^{3} P\left(\frac{E_i}{E}\right) = P\left(\frac{E_1}{E}\right) + P\left(\frac{E_2}{E}\right) + P\left(\frac{E_3}{E}\right)$$

$$= \frac{P(E_1) \times P\left(\frac{E}{E_1}\right) + P(E_2) \times P\left(\frac{E}{E_2}\right) + P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right) + P(E_3)\left(\frac{E}{E_3}\right)}$$

$= 1$

Very Short Answer Type Questions

Q 1. Let A and B be two events such that $P(A)=\frac{5}{8}, P(B)=\frac{1}{2}$ and $P(A/B)=\frac{3}{4}$. Find the value of $P(B/A)$. (CBSE 2022 Term-2)

Q 2. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die. (CBSE 2022 Term-2)

Q 3. The probabilities of A and B solving a problem independently are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If both of them try to solve the problem independently. What is the probability that the problem is solved?

Q 4. The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probability that the target is hit. (CBSE 2022 Term-2)

Q 5. If E and F are two events such that $P(E)=0.6$, $P(F)=0.3$ and $P(E\cap F)=0.2$. Find the values of $P(E/F)$ and $P(F/E)$. (NCERT EXERCISE)

Q 6. Find $P(A\cup B)$ if $2P(A)=P(B)=\frac{5}{13}$ and $P\left(\frac{A}{B}\right)=\frac{2}{5}$.

Q 7. A and B are two independent events. If $P(A)=0.3$, $P(B)=0.4$, then find (i) $P(A\cap B)$, (ii) $P(A\cup B)$.

Q 8. Suppose A and B are independent events, such that $P(A)=0.3$ and $P(B)=0.4$, then find $P\left(\frac{A}{B}\right)$. (NCERT EXERCISE)

Q 9. If $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{2}{3}$. Are the events A and B independent?

Q 10. If A and B are two independent events then prove that the probability of occurrence of at least one of A and B is given by $1-P(A')\,P(B')$. (NCERT EXERCISE)

Q 11. Two cards are drawn at random from a pack of 52 cards one-by-one without replacement. What is the probability of getting first card red and second card Jack? (CBSE SQP 2022 Term-2)

Q 12. Three distinct numbers are chosen randomly from the first 50 natural numbers. Find the probability that all the three numbers are divisible by both 2 and 3. (CBSE 2020)

Q 13. If P (not A) $=0.7$, $P(B)=0.7$ and $P(B/A)=0.5$, then find $P(A/B)$. (CBSE 2019)

Q 14. If $P(A)=0.4$, $P(B)=p$, $P(A\cup B)=0.6$ and A and B are given to be independent events, find the value of 'p'. (CBSE 2017)

Short Answer Type-I Questions

Q 1. A room has three lamp sockets. From a collection of 10 light bulbs of which only 6 are defective, three bulbs are selected at random and placed in the socket. What is the probability that there will be light in the room?

Q 2. Find $[P(B/A)+P(A/B)]$, if $P(A)=\frac{3}{10}$, $P(B)=\frac{2}{5}$ and $P(A\cup B)=\frac{3}{5}$. (NCERT EXEMPLAR; CBSE 2020)

Q 3. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4. (CBSE 2018)

Q 4. Ten cards numbered 1 to 10 are placed in a box after thoroughly mixing. One card is drawn from the box at random. If it is known that the number on the card is more than 3 then find the probability that this number is even number.

Q 5. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy, (ii) the older child is a boy.

Q 6. From a pack of 52 playing cards, two are drawn one by one. After drawing they are not returned to the pack again, then find the probability of both cards to the betel.

Q 7. Given two independent events A and B such that $P(A)=0.3$ and $P(B)=0.6$, find $P(A'\cap B')$. (CBSE 2020)

Q 8. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not. (CBSE 2019, 17)

Q 9. Prove that if E and F are independent events, then the events E and F' are also independent. (NCERT EXERCISE; CBSE 2017)

Q 10. A can hit a target 4 times in 5 shots, B 3 times in 4 shots and C 2 times in 3 shots. Find the probability that target will be hit.

Q 11. There are 4 red and 5 black balls in a bag A. In another bag B there are 6 red and 3 black balls. One red ball is taken from bag A and transferred to bag B. After this one ball is taken from bag B, find the probability of that to be red.

Q 12. A speaks truth in 80% cases and B speaks truth in 90% cases. In what percentage of cases are they likely to agree with each other in stating the same fact?

Q 13. Three persons A, B and C, fire at a target in turn, starting with A. Their probability of hitting the target are 0.4, 0.3 and 0.2 respectively. Find the probability of two hits. *(NCERT EXEMPLAR)*

Q 14. A refrigerator box contains 2 milk chocolates and 4 dark chocolates. Two chocolates are drawn at random. Find the probability distribution of the number of milk chocolates. What is the most likely outcome?

Q 15. A bag contains 1 red and 3 white balls. Find the probability distribution of the number of red balls if 2 balls are drawn at random from the bag one-by-one without replacement. *(CBSE SQP 2022 Term-2)*

Q 16. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability of X. *(CBSE 2022 Term-2)*

Q 17. Two Cards are drawn successively with replacement from a well shuffled pack of 52 Cards. Find the probability distribution of the number of spade cards. *(CBSE 2022 Term-2)*

Q 18. A biased die is such that $P(4)=\frac{1}{10}$ and other scores are equally likely. The die is tossed twice. If X is the 'number of four obtained', find the mean of X.

Q 19. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean of X. *(CBSE 2017)*

Q 20. Find the mean of the number obtained on a throw of an unbiased die. *(NCERT EXERCISE)*

Short Answer Type-II Questions

Q 1. There are 4 white and 3 black balls in a bag. In these ball drawn one by one without replacement then find the probability that first ball is white, second ball is black, third ball is white, fourth ball is black, fifth ball is white, sixth ball is black and seventh ball is white.

Q 2. In a class 40% students read Mathematics, 20% read Biology and 10% read both Mathematics and Biology. One student is selected at random. Find the probability that:

(i) he reads Mathematics, when it is know that he reads Biology.

(ii) he reads Biology when it is known that he reads Mathematics.

Q 3. From integers 1 to 11 two integers are selected at random. If their sum is even, then find the probability that both integers are odd.

Q 4. Three friends go for coffee. They decide who will pay the bill, by each tossing a coin and then letting the 'odd person' pay. There is no odd person if all three tosses produce the same result. If there is no odd person in the first round, they make a second round of tosses and they continue to do so until there is an odd person. What is the probability that exactly three rounds of tosses are made? *(CBSE SQP 2022-23)*

Q 5. Suppose that 5 men out of 100 and 25 women out of 1000 are good orators. Assuming that there are equal number of men and women, find the probability of choosing a good orator.

Q 6. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, then find the probability of one of them being red and another black.

Q 7. A bag contains $(2n+1)$ coins. It is known that $(n-1)$ of these coins have a head on both sides, whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n. *(NCERT EXEMPLAR)*

Q 8. Three machines E_1, E_2 and E_3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E_1 and E_2 are defective and that 5% of those produced by machine E_3 are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective. *(NCERT EXEMPLAR)*

Q 9. In a bag there are 5 red, 4 black and 3 white balls. If one by one three balls are taken out and not again returned to the bag, then find the probability that all the three balls drawn are red.

Q 10. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace? *(NCERT EXERCISE)*

Q 11. A and B are two independent events. Prove that:

(i) A' and B' will also be independent events

(ii) $P(B/A)=P(B/A')=P(B)$ *(NCERT EXEMPLAR)*

Q 12. A man is known to speak truth 7 out of 10 times. He threw a pair of dice and reports that doublet appeared. Find the probability that it was actually a doublet. *(CBSE 2022 Term-2)*

Q 13. A person A speaks truth in 70% of the cases while second person B speaks truth in 80% of the cases. Find the probability that in what percentage of cases they will:

(i) agree (ii) oppose

to each other, for the same statement of fact.

Q 14. A and B throw a pair of dice alternately. A wins the game if he gets a total of 9 and B wins if he gets a total of 7. If A starts the game, find the probability of winning the game by B.

Q 15. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die? *(CBSE 2018)*

Q 16. In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y. *(CBSE 2020, 17)*

Q 17. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer. *(CBSE 2017)*

Q 18. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items further 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine A? *(NCERT EXERCISE)*

Q 19. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. *(CBSE 2017)*

Q 20. Given three identical boxes I, II and III, each containing two coins. In box I, both are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If coin is of gold, what is the probability that the other coin in the box is also of gold? *(NCERT EXERCISE)*

Q 21. A patient travels to a doctor by train, bus, scooter or any other means of transport whose probabilities are $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ or $\frac{2}{5}$ respectively. If he travels by train, bus or scooter, he reaches late and their probabilities are $\frac{1}{4}, \frac{1}{3}$ or $\frac{1}{12}$ respectively. He reaches on time if he travels by any other means of transport. If he reached late, find the probability of his coming by train.

Q 22. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some real number:

$$P(X)=\begin{cases} k, \text{ if } x=0 \\ 2k, \text{ if } x=1 \\ 3k, \text{ if } x=2 \\ 0, \text{ otherwise} \end{cases}$$

(i) Determine the value of k. *(CBSE 2019)*

(ii) Find $P(X<2)$

(iii) Find $P(X>2)$ *(CBSE SQP 2023-24)*

Q 23. The random variable X can take only the values 0, 1, 2, 3. Given that $P(2)=P(3)=p$ and $P(0)=2P(1)$. If $\Sigma P_i x_1^2 = 2\Sigma P_i x_i$, find the value of p. *(CBSE 2017)*

Q 24. A coin is biased so that the head is three times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. Hence, find the mean of the number of tails.

Q 25. Find the mean number of defective items in a sample of two items drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items. Assume that the items are identical in shape and size. *(CBSE SQP 2022-23)*

Q 26. From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs. *(CBSE 2023)*

Q 27. Two fair dice are thrown simultaneously. If X denotes the number of sixes, find the mean of X. *(CBSE 2023)*

Q 28. Two numbers are selected at random (without replacement) from the first five positive integers. Let X denotes the larger of the two numbers obtained. Find the mean of X. *(CBSE 2018)*

Q 29. There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean of X.

Long Answer Type Questions

Q 1. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly. *(CBSE 2023)*

Q 2. A and B are two independent events. The probability of their simultaneously occurrence is $\frac{1}{6}$ and the probability of their simultaneously non-occurrence is $\frac{1}{3}$. Find the probabilities of the occurrence of A and B separately.

Q 3. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A? *(CBSE 2019)*

Q 4. An urn contains 4 balls. Two balls are drawn at random from the urn (without replacement) and are found to be white. What is the probability that all the four balls in the urn are white.

Q 5. A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize. *(CBSE 2023)*

Q 6. Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X. *(NCERT EXERCISE)*

Q 7. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean of the number of kings. *(CBSE 2019)*

Q 8. Three numbers are selected at random (without replacement) from first six positive integers. If X denotes the smallest of the three numbers obtained, find the probability distribution of X. Also, find the mean of the distribution.

Solutions

Very Short Answer Type Questions

1. Given, $P(A) = \frac{5}{8}$ and $P(B) = \frac{1}{2}$.

Also, $P(A/B) = \frac{3}{4}$

$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} = \frac{3}{4}$

$\Rightarrow \quad \frac{P(A \cap B)}{1/2} = \frac{3}{4}$

$\Rightarrow \quad P(A \cap B) = \frac{3}{8}$.

$\therefore \quad P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{3}{8} \times \frac{8}{5} = \frac{3}{5}$

2. Total ways for numbers on the die for the sum to be 7
$= \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$

Thus, there are 6 ways possible for the sum to be 7.

Here, favourable cases $= \{(2, 5), (5, 2)\}$

So, out of these six ways, there are 2 ways where 5 is one of the outcome.

Thus, required probability $= \frac{2}{6} = \frac{1}{3}$.

3. Given, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

$\Rightarrow \quad P(A') = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

and $\quad P(B') = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$

TR!CK

The probability that the problem is solved = 1 − Probability that none of them solved the problem.

$\therefore$ Required probability $= 1 - P(A') \cdot P(B')$

$= 1 - \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{2} = \frac{1}{2}$

4. Given, $P(A) = P$ (A hits the target) $= \frac{1}{3}$.

$P(B) = P$ (B hits target) $= \frac{2}{5}$.

Now, $P(A \cup B) = P$ (target will be hit by either A or B)

$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

[$\because A$ and B are independent]

$= \frac{1}{3} + \frac{2}{5} - \frac{1}{3} \times \frac{2}{5} = \frac{5+6-2}{15} = \frac{9}{15} = \frac{3}{5}$.

5. $P(E/F)=\dfrac{P(E\cap F)}{P(F)}=\dfrac{0.2}{0.3}=\dfrac{2}{3}$

and $P(F/E)=\dfrac{P(F\cap E)}{P(E)}=\dfrac{0.2}{0.6}=\dfrac{1}{3}$

6. Given, $2P(A)=P(B)=\dfrac{5}{13}$

$\Rightarrow$ $P(A)=\dfrac{5}{26}$

and $P(B)=\dfrac{5}{13}$ and $P\left(\dfrac{A}{B}\right)=\dfrac{2}{5}$

$\therefore$ $P(A\cap B)=P\left(\dfrac{A}{B}\right)\cdot P(B)=\dfrac{2}{5}\cdot\dfrac{5}{13}=\dfrac{2}{13}$

$\therefore$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\dfrac{5}{26}+\dfrac{5}{13}-\dfrac{2}{13}=\dfrac{5+10-4}{26}=\dfrac{11}{26}$

7. (i) A and B are two independent events

$\therefore$ $P(A\cap B)=P(A)\cdot P(B)=0.3\times 0.4=0.12$

(ii) $P(A\cup B)=P(A)-P(B)-P(A\cap B)$

$=0.3+0.4-0.12=0.58$

8. Given, $P(A)=0.3$ and $P(B)=0.4$

When A and B are independent events,

$P(A\cap B)=P(A)\times P(B)=0.3\times 0.4=0.12$

$\therefore$ $P\left(\dfrac{A}{B}\right)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.12}{0.4}=\dfrac{3}{10}$

9.

TiP

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore$ $P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{2}{3}=\dfrac{3+2-4}{6}=\dfrac{1}{6}$

while $P(A)\cdot P(B)=\dfrac{1}{2}\times\dfrac{1}{3}=\dfrac{1}{6}$,

i.e., $P(A)\cdot P(B)=P(A\cap B)$

$\therefore$ Events A and B are independent.

10. Probability of non-occurrence of any of the events A and B $=P(A'\cap B')$

$=P(\overline{A\cup B})=1-P(A\cup B)$

$=1-\{P(A)+P(B)-P(A\cap B)\}$

$=1-\{P(A)+P(B)-P(A)\cdot P(B)\}$

[$\because$ A and B are independent events]

$=1-P(A)-P(B)\{1-P(A)\}$

$=(1-P(A))(1-P(B))$

$=P(A')P(B')$

Therefore, the probability of occurrence of atleast one of A or $B=1-P(A')P(B')$. **Hence proved.**

11. The required probability

$=P$ [(The first is a red jack card and the second is a jack card) or (The first is a red non-jack card and the second is a jack-card)]

$=\dfrac{2}{52}\times\dfrac{3}{51}+\dfrac{24}{52}\times\dfrac{4}{51}=\dfrac{1}{26}$

12. There are only 8 numbers in first 50 natural numbers, which are divisible both 2 and 3, *i.e.*, 6.

6, 12, 18, ..., 48

TR!CK

A number divisible by both 2 and 3 i.e., LCM (2, 3) = 6

Now, total number of possible outcomes $={}^{50}C_3$

$\therefore$ Favourable number of outcomes $={}^{8}C_3$

So, required probability $=\dfrac{{}^{8}C_3}{{}^{50}C_3}$

$=\dfrac{\dfrac{8\times7\times6}{3\times2\times1}}{\dfrac{50\times49\times48}{3\times2\times1}}=\dfrac{1}{350}$

13. Given, P (not A) $=0.7=P(\overline{A})$ and $P(B)=0.7$

$\therefore$ $P(A)=1-P(\overline{A})=1-0.7=0.3$

Also given, $P(B/A)=0.5$

$\therefore$ $P(A\cap B)=P(A)\cdot P(B/A)=0.3\times0.5=0.15$

So, $P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{0.15}{0.7}=\dfrac{15}{70}=\dfrac{3}{14}$

14. Given, $P(A)=0.4$, $P(B)=p$ and $P(A\cup B)=0.6$

$\because$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

[By addition theorem of probability]

$\Rightarrow$ $P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=0.4+p-0.6=p-0.2$

Since, A and B are independent events.

TiP

The events are independent if and only if $P(A\cap B)=P(A)\cdot P(B)$.

$\therefore$ $P(A)\cdot P(B)=P(A\cap B)$

$\Rightarrow$ $0.4\times p=p-0.2$

$\Rightarrow$ $p-0.4p=0.2$

$\Rightarrow$ $0.6p=0.2$

$\therefore$ $p=\dfrac{0.2}{0.6}=\dfrac{1}{3}$

Short Answer Type-I Questions

1. Number of bulbs = 10

Number of selection of 3 bulbs out of 10 $={}^{10}C_3$

Now, given that 6 bulbs are defective, then 4 bulbs are good.

Probability that there will be no light in the room

= Probability of selection of three defective bulbs

$=\dfrac{{}^{6}C_3}{{}^{10}C_3}=\dfrac{\dfrac{6!}{3!3!}}{\dfrac{10!}{3!7!}}=\dfrac{6!}{3!3!}\times\dfrac{3!7!}{10!}$

$=\dfrac{6\times5\times4}{3\times2}\times\dfrac{3\times2}{10\times9\times8}=\dfrac{1}{6}$

$\therefore$ Probability that there will be light in the room

$=1-$ Probability that there will be no light in the room

$=1-\dfrac{1}{6}=\dfrac{5}{6}$

2. Given, $P(A) = \frac{3}{10} = 0.3, P(B) = \frac{2}{5} = 0.4$

and $P(A \cup B) = \frac{3}{5} = 0.6$

$\because$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore$ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.3 + 0.4 - 0.6 = 0.7 - 0.6 = 0.1$

Now, $P(B/A) + P(A/B) = \frac{P(B \cap A)}{P(A)} + \frac{P(A \cap B)}{P(B)}$

$= \frac{0.1}{0.3} + \frac{0.1}{0.4} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

3. Let the first observation be from the black die and second from the red die.

When two dice (one black and another red) are rolled, the sample space (S) has $6 \times 6 = 36$ number of elements.

E : Sum of the observation is 8.

$= \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

F : Red die resulted in a number less than 4

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

$\therefore$ $E \cap F = \{(5, 3), (6, 2)\}$

Now, $P(F) = \frac{18}{36} = \frac{1}{2}$

and $P(E \cap F) = \frac{2}{36} = \frac{1}{18}$

The conditional probability of obtaining the sum equal to 8, given that the red die resulted in a number less than 4, is given by $P(E/F)$.

Therefore, $P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{2/36}{1/2} = \frac{2}{36} \times 2 = \frac{1}{9}$

COMMON ERROR

Students get confused in deciding whether to find P (A/B) or P (B/A). So, more practice is required.

4. Let E = event that the number on the card is more than 3

TiP

Learn the concepts of conditional probability and its applications.

and F = event that the number on the card is even.

Here, $n(S) = 10$

$E = \{4, 5, 6, 7, 8, 9, 10\} \Rightarrow n(E) = 7$

$F = \{2, 4, 6, 8, 10\} \Rightarrow n(F) = 5$

$E \cap F = \{4, 6, 8, 10\} \Rightarrow n(E \cap F) = 4$

Now, $P(F/E)$ = probability that the number is even when it is known that it is more than 3.

$= \frac{P(F \cap E)}{P(E)} = \frac{4/10}{7/10} = \frac{4}{7}$

5. Given, a couple has 2 children, then sample space $(S) = \{B_1B_2, B_1G_2, G_1B_2, G_1G_2\}$ where B_1 and G_1 are the older boy and girl respectively.

Let E_1 = both the children are boys.

E_2 = one of the children is a boy.

E_3 = the older child is a boy.

$P(E_2) = \frac{3}{4}, P(E_3) = \frac{2}{4},$

$P(E_1 \cap E_2) = \frac{1}{4}$ and $P(E_1 \cap E_3) = \frac{1}{4}$

(i) $P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/4}{3/4} = \frac{1}{3}$

(ii) $P(E_1/E_3) = \frac{P(E_1 \cap E_3)}{P(E_3)} = \frac{1/4}{2/4} = \frac{1}{2}$

6. Let, Event A: first card of betel

Event B: second card of betel

Probability of getting first card of betel,

$P(A) = \frac{13}{52} = \frac{1}{4}$ ($\because$ Number of betel cards = 13)

Remaining cards; betel cards = 12, total cards = 51

$\therefore$ Probability of getting second card of betel,

$P\left(\frac{B}{A}\right) = \frac{12}{51} = \frac{4}{17}$

Hence, required probability (both cards to be betel)

$P(A \cap B) = P(A) \cdot P\left(\frac{B}{A}\right) = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$

7. Given that, $P(A) = 0.3$ and $P(B) = 0.6$

Since, A and B are two independent events.

TR!CKS

- *By Demorgan's law,* $(\overline{A \cup B}) = \bar{A} \cap \bar{B}, (\overline{A \cap B}) = \bar{A} \cup \bar{B}.$
- $P(E) + P(\bar{E}) = 1$
- *By addition theorem of probability,* $P(A \cup B) = P(A) + P(B) - P(A \cap B).$

$\therefore$ $P(A \cap B) = P(A) \cdot P(B) = 0.3 \times 0.6 = 0.18$

So, $P(A' \cap B') = 1 - P(\overline{A' \cap B'}) = 1 - P(A \cup B)$

$= 1 - \{P(A) + P(B) - P(A \cap B)\}$

$= 1 - (0.3 + 0.6 - 0.18) = 1 - (0.9 - 0.18)$

$= 1 - 0.9 + 0.18 = 1.18 - 0.9 = 0.28$

8. When a die is thrown, the sample space is:

$S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

Also, A: Number is even and B: number is red.

$\therefore$ $A = \{2, 4, 6\}, B = \{1, 2, 3\}$

and $A \cap B = \{2\}$

$\Rightarrow$ $n(A) = 3, n(B) = 3$

and $n(A \cap B) = 1$

Now, $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

and $P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$

TR!CKS

- *If E and F are independents events, then*
  $P(E \cap F) = P(E) \cdot P(F)$
- *If E and F are dependents events, then*
  $P(E \cap F) \neq P(E) \cdot P(F)$

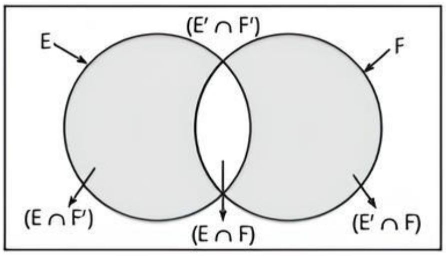
Now, $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq \frac{1}{6} = P(A \cap B)$

$\therefore \quad P(A \cap B) \neq P(A) \cdot P(B)$

Thus, A and B are not independent events.

9. Since, E and F are independent events.

$\therefore \quad P(E \cap F) = P(E) \cdot P(F) \quad \text{...(1)}$



It is clear from the following Venn diagram that events $E \cap F$ and $E \cap F'$ are mutually exclusive and

$$E = (E \cap F) \cup (E \cap F')$$

$\because$ $E \cap F$ and $E \cap F'$ are mutually exclusive.

$\therefore \quad P(E) = P(E \cap F) + P(E \cap F')$

$\Rightarrow \quad P(E \cap F') = P(E) - P(E \cap F)$

$= P(E) - P(E) \cdot P(F)$ [From eq. (1)]

$= P(E)\{1 - P(F)\} = P(E) \cdot P(F')$

$[\because P(F) + P(F') = 1]$

Hence, E and F' are independent events.

Hence proved.

10. Hitting the target means that A, B and C all trying to hit the target together.

Given, A can hit a target 4 times in 5 shots

$\therefore$ Probability to hit the target by $A = \frac{4}{5} = P(A)$

B can hit a target 3 times in 4 shots.

$\therefore$ Probability to hit the target by $B = \frac{3}{4} = P(B)$

C can hit a target 2 times in 3 shots.

$\therefore$ Probability to hit the target by $C = \frac{2}{3} = P(C)$.

So, required probability to hit the target

$= 1 - \{1 - P(A)\}\{1 - P(B)\}\{1 - P(C)\}$

$= 1 - \left(1 - \frac{4}{5}\right)\left(1 - \frac{3}{4}\right)\left(1 - \frac{2}{3}\right)$

$= 1 - \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}$

$= 1 - \frac{1}{60} = \frac{60 - 1}{60} = \frac{59}{60}$

COMMON ERR(!)R

Mostly students find $P(A \cap B \cap C)$ instead of finding $P(A \cup B \cup C)$.

11. Here, $R_1 = \{4 \text{ red balls}\}$

$B_1 = \{5 \text{ black balls}\}$

$R_2 = \{6 \text{ red balls}\}$

and $B_2 = \{3 \text{ black balls}\}$

Let event E_1 = one red ball is taken from first bag and transferred to second bag.

Event E_2 = one black ball is taken from first bag and transferred to second bag.

and event E = a red ball drawn randomly from second bag

$\therefore \quad P\left(\frac{E}{E_1}\right) = \frac{7}{10}, \quad P\left(\frac{E}{E_2}\right) = \frac{6}{10}$

and $\quad P(E_1) = \frac{4}{9}, \quad P(E_2) = \frac{5}{9}$

Hence, $\quad P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$

$= \frac{4}{9} \times \frac{7}{10} + \frac{5}{9} \times \frac{6}{10}$

$= \frac{28}{90} + \frac{30}{90} = \frac{58}{90} = \frac{29}{45}$

COMMON ERR(!)R

Mostly students have a confusion whether to apply total probability theorem or Baye's theorem.

12. Here, $P(A) = \frac{80}{100} = \frac{4}{5}$ and $P(B) = \frac{90}{100} = \frac{9}{10}$

$\therefore P$ (Agree) $= P$ (both speaking truth or both telling lie)

$= P(AB \text{ or } \overline{A}\overline{B}) = P(A)P(B) \text{ or } P(\overline{A})P(\overline{B})$

$= \frac{4}{5} \times \frac{9}{10} + \{1 - P(A)\}\{1 - P(B)\}$

$= \frac{18}{25} + \left(1 - \frac{4}{5}\right)\left(1 - \frac{9}{10}\right) = \frac{18}{25} + \frac{1}{5} \times \frac{1}{10}$

$= \frac{18}{25} + \frac{1}{50} = \frac{36 + 1}{50} = \frac{37}{50} = \frac{74}{100} = 74\%$

13. Here, $P(A) = 0.4$, $P(B) = 0.3$ and $P(C) = 0.2$

$\therefore \quad P(\overline{A}) = 1 - P(A), P(\overline{B}) = 1 - P(B)$

$P(\overline{A}) = 1 - P(A) = 1 - 0.4 = 0.6,$

$P(\overline{B}) = 1 - 0.3 = 0.7, P(\overline{C}) = 1 - 0.2 = 0.8$

So, probability of two hits $= P(A) \cdot P(B) \cdot P(\overline{C})$

$+ P(A) \cdot P(\overline{B}) \cdot P(C) + P(\overline{A}) \cdot P(B) \cdot P(C)$

$= (0.4)(0.3)(0.8) + (0.4)(0.7)(0.2) + (0.6)(0.3)(0.2)$

$= 0.096 + 0.056 + 0.036 = 0.188$

COMMON ERR(!)R

Students face problems in deciding when to add or when to multiply probability. So, adequate practice is required.

14. Let X denotes the number of milk chocolate drawn.

So, X can be 0, 1 and 2.

$P(X = 0) = \frac{4}{6} \times \frac{3}{5} = \frac{12}{30},$

$P(X = 1) = \left(\frac{2}{6} \times \frac{4}{5}\right) \times 2 = \frac{16}{30},$

$P(X = 2) = \frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$

∴ Required probability distribution is:

X	0	1	2
$P(X)$	$\frac{12}{30}$	$\frac{16}{30}$	$\frac{2}{30}$

Most likely outcome is getting one chocolate of each type.

COMMON ERR!OR

Students get confused whether the items are drawn with replacement or without replacement.

15. Let X be the random variable defined as the number of red balls.

Then $X = 0, 1$

$$P(X=0) = \frac{3}{4}\times\frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

$$P(X=1) = \frac{1}{4}\times\frac{3}{3} + \frac{3}{4}\times\frac{1}{3} = \frac{6}{12} = \frac{1}{2}$$

∴ Probability distribution table is:

X	0	1
$P(X)$	1/2	1/2

16. Given, the variable X denotes the number of red balls.

Then, $X = 0, 1, 2$

$$P(X=0) = \frac{3}{5}\times\frac{2}{4} = \frac{3}{10}.$$

$$P(X=1) = \frac{2}{5}\times\frac{3}{4} + \frac{3}{5}\times\frac{2}{4} = \frac{3}{10} + \frac{3}{10}$$

$$= \frac{6}{10} = \frac{3}{5}$$

$$P(X=2) = \frac{2}{5}\times\frac{1}{4} = \frac{1}{10}$$

∴ Probability distribution table is:

X	0	1	2
$P(X)$	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{1}{10}$

17. Let X denotes the number of spade cards drawn.

Then the value of X will be 0, 1 and 2.

Now, the probability of drawing a spade card $= \frac{13}{52} = \frac{1}{4}$

and the probability of not drawing a spade card $= 1 - \frac{1}{4} = \frac{3}{4}$.

∴ $$P(X=0) = \frac{3}{4}\times\frac{3}{4} = \frac{9}{16}$$

$$P(X=1) = \frac{1}{4}\times\frac{3}{4} + \frac{3}{4}\times\frac{1}{4}$$

$$= \frac{3}{16} + \frac{3}{16} = \frac{6}{16} = \frac{3}{8}$$

$$P(X=2) = \frac{1}{4}\times\frac{1}{4} = \frac{1}{16}$$

So, the probability distribution of the number of spade cards is:

X	0	1	2
$P(X)$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

18. Here, X is the number of fours obtained. So, X can take values 0, 1 and 2.

∴ Probability distribution is:

X	0	1	2
$P(X)$	$\left(\frac{9}{10}\right)^2 = \frac{81}{100}$	$2\times\frac{9}{10}\times\frac{1}{10} = \frac{18}{100}$	$\left(\frac{1}{10}\right)^2 = \frac{1}{100}$

Now, Mean $\mu = \Sigma X P(X) = X_0P_0 + X_1P_1 + X_2P_2$

$$= 0\times\frac{81}{100} + 1\times\frac{18}{100} + 2\times\frac{1}{100} = \frac{20}{100} = \frac{2}{10}$$

19. Given, X denotes the sum of the numbers.

So, X can be 4, 6, 8, 10, 12.

X	Number on card	$P(X)$	$XP(X)$
4	(1, 3)	$\frac{1}{4}\times\frac{1}{3}\times 2 = \frac{1}{6}$	$\frac{2}{3}$
6	(1, 5)	$\frac{1}{4}\times\frac{1}{3}\times 2 = \frac{1}{6}$	1
8	(3, 5) or (1, 7)	$\frac{1}{4}\times\frac{1}{3}\times 2 + \frac{1}{4}\times\frac{1}{3}\times 2 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$	$\frac{8}{3}$
10	(3, 7)	$\frac{1}{4}\times\frac{1}{3}\times 2 = \frac{1}{6}$	$\frac{5}{3}$
12	(5, 7)	$\frac{1}{4}\times\frac{1}{3}\times 2 = \frac{1}{6}$	2

TR!CK

Let X be a random variable taking values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively. Then mean of a random variable X is $\sum_{i=1}^{n} x_i p_i$.

∴ Mean $\Sigma XP(X) = \frac{2}{3} + 1 + \frac{8}{3} + \frac{5}{3} + 2 = \frac{15}{3} + 3 = 5 + 3 = 8$

20. Sample space of test: $S = \{1, 2, 3, 4, 5, 6\}$

Let X be the number that appear on the die, then X is a random variable, which can take the values 1, 2, 3, 4, 5, or 6.

$$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$$

∴ Probability distribution of X is as following:

X	1	2	3	4	5	6
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Now, Mean $E(X) = \sum_{i=1}^{n} x_i P(x_i)$

$$= 1\times\frac{1}{6} + 2\times\frac{1}{6} + 3\times\frac{1}{6} + 4\times\frac{1}{6} + 5\times\frac{1}{6} + 6\times\frac{1}{6}$$

$$= \frac{21}{6}$$

Short Answer Type-II Questions

1. Number of white balls = 4

and Number of black balls = 3

$\therefore$ Number of balls in a bag = 4 + 3 = 7

$\because$ All these balls drawn one by one without repetition.

$\therefore$ Probability of getting first ball is white $= \frac{4}{7}$

Probability of getting second ball is black

$$= \frac{3}{(7-1)} = \frac{3}{6} = \frac{1}{2}$$

Probability of getting third ball is white $= \frac{(4-1)}{(6-1)} = \frac{3}{5}$

Probability of getting fourth ball is black

$$= \frac{(3-1)}{(5-1)} = \frac{2}{4} = \frac{1}{2}$$

Probability of getting fifth ball is white $= \frac{(3-1)}{(4-1)} = \frac{2}{3}$

Probability of getting sixth ball is black $= \frac{(2-1)}{(3-1)} = \frac{1}{2}$

Probability of getting seventh ball is white $= \frac{2-1}{2-1} = \frac{1}{1} = 1$

So, required probability $= \frac{4}{7} \times \frac{1}{2} \times \frac{3}{5} \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \times 1 = \frac{1}{35}$

2. Let M = event of the group of students read mathematics and B = event of the group of students read biology.

Suppose $n(S) = 100$,

then $n(M) = 40$, $n(B) = 20$

and $n(M \cap B) = 10$

$\therefore$ $P(M) = \frac{n(M)}{n(S)} = \frac{40}{100} = \frac{2}{5}$

TiP

Learn the concepts of conditional probability and its applications.

$$P(B) = \frac{n(B)}{n(S)} = \frac{20}{100} = \frac{1}{5}$$

and $P(M \cap B) = \frac{n(M \cap B)}{n(S)} = \frac{10}{100} = \frac{1}{10}$

(i) Required probability = P (he reads mathematics, when it is known that he reads biology)

$$\therefore P\left(\frac{M}{B}\right) = \frac{P(M \cap B)}{P(B)} = \frac{\frac{1}{10}}{\frac{1}{5}} = \frac{5}{10} = \frac{1}{2}$$

(ii) Required probability = P (he read biology when it is known that he reads Mathematics)

$$\therefore P\left(\frac{B}{M}\right) = \frac{P(M \cap B)}{P(M)} = \frac{\frac{1}{10}}{\frac{2}{5}} = \frac{5}{2} \times \frac{1}{10} = \frac{1}{4}$$

3. Let event A = sum of the numbers is even.

TiP

Practice more problems on elementary probability, involving combinations.

and event B = both numbers are odd.

From integers 1 to 11, the sum will be even only when either both the numbers are odd or both are even. There are 6 odd numbers and 5 even numbers.

$$\therefore \quad P(A) = \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_2}$$

and $$P(B) = \frac{{}^6C_2}{{}^{11}C_2} = P(A \cap B)$$

Hence, required probability $= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$$= \frac{\frac{{}^6C_2}{{}^{11}C_2}}{\frac{{}^6C_2 + {}^5C_2}{{}^{11}C_2}} = \frac{{}^6C_2}{{}^6C_2 + {}^5C_2} = \frac{\frac{6!}{2!4!}}{\frac{6!}{2!4!} + \frac{5!}{2!3!}}$$

$$= \frac{15}{15+10} = \frac{15}{25} = \frac{3}{5}$$

COMMON ERROR

Mostly students go wrong in problems involving permutations and combinations.

4. P (not obtaining an odd person in a single round)

$= P$ (All three of them throw tails or all three of them throw heads)

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{4}$$

and P (obtaining an odd person in a single round)

$= 1 - P$ (not obtaining an odd person in a single round)

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ Required probability

$= P$ ('In first round there is no odd person' and 'In second round there is no odd person' and 'In third round there is an odd person')

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{64}$$

5. Let E_1 and E_2 denote the events that the person is a good orator. E_1 = Man orator and E_2 = Woman orator

$\therefore$ $P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{2}$

Also let, A = Selecting good orator

$$P(A/E_1) = \frac{5}{100}$$

and $$P(A/E_2) = \frac{25}{1000}$$

So, P (choosing good orator) $= P(A)$

$$= P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)$$

$$= \frac{1}{2} \times \frac{5}{100} + \frac{1}{2} \times \frac{25}{1000} = \frac{75}{2000} = \frac{3}{80}$$

6. Let E_1 = Event selecting bag A, E_2 = Event selecting bag B

TiP

Total probability theorem gives the total probability of an event whereas the Baye's theorem is basically a conditional probability.

and Event A = Getting one red and one black ball.

Here, $P(E_1)=\frac{1}{3}$ and $P(E_2)=\frac{2}{3}$

Also, $P(A/E_1)=\frac{^4C_1\cdot {^6C_1}}{^{10}C_2}$

$=\frac{4\times 6}{5\times 9}=\frac{8}{15}$

and $P(A/E_2)=\frac{^7C_1\cdot {^3C_1}}{^{10}C_2}=\frac{7\times 3}{5\times 9}=\frac{7}{15}$

$\therefore$ The probability that getting one red and one black ball,

$$P(A)=P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)$$
$$=\frac{1}{3}\times\frac{8}{15}+\frac{2}{3}\times\frac{7}{15}=\frac{22}{45}$$

COMMON ERROR

Mostly students have a confusion whether to apply total probability theorem or Baye's theorem.

7. Given, number of coins with head on both sides $=(n-1)$

$\therefore$ Number of fair coins $=(2n+1)-(n-1)=n+2$

Let event

E_1 = Picking a coin with head on both sides

E_2 = Picking a fair coin

A = Getting a head on tossing the coin

$\therefore$ $P(E_1)=\frac{n-1}{2n+1}, P(E_2)=\frac{n+2}{2n+1}$

$P\left(\frac{A}{E_1}\right)=1$ and $P\left(\frac{A}{E_2}\right)=\frac{1}{2}$

Now, $P(A)=P(E_1)P(A/E_1)+P(E_2)P(A/E_2)$

$=\frac{(n-1)}{(2n+1)}\times 1+\frac{(n+2)}{(2n+1)}\times\frac{1}{2}=\frac{3n}{2(2n+1)}$

$\because$ The probability that the toss results in a head $=\frac{31}{42}$

$\therefore$ $\frac{3n}{2(2n+1)}=\frac{31}{42}$

$\Rightarrow$ $63n=62n+31$

$\Rightarrow$ $n=31$

COMMON ERROR

Mostly students have a confusion whether to apply total probability theorem or Baye's theorem.

8. Let A_1: Event that the bulb is produced by machine E_1.

A_2: Event that the bulb is produced by machine E_2.

A_3: Event that the bulb is produced by machine E_3.

A: Event that the picked up bulb is defective.

TiP

Total probability theorem gives the total probability of an event whereas the Baye's theorem is basically a conditional probability.

Here, $P(A_1)=50\%=\frac{50}{100}=\frac{1}{2}$,

$P(A_2)=25\%=\frac{25}{100}=\frac{1}{4}$,

$P(A_3)=25\%=\frac{25}{100}=\frac{1}{4}$.

Also, $P\left(\frac{A}{A_1}\right)=4\%=\frac{4}{100}=\frac{1}{25}$

$P\left(\frac{A}{A_2}\right)=4\%=\frac{4}{100}=\frac{1}{25}$

and $P\left(\frac{A}{A_3}\right)=5\%=\frac{5}{100}=\frac{1}{20}$

$\therefore$ The probability that the picked bulb is defective,

$P(A)$

$=P(A_1)\times P\left(\frac{A}{A_1}\right)+P(A_2)\times P\left(\frac{A}{A_2}\right)+P(A_3)\times P\left(\frac{A}{A_3}\right)$

$=\frac{1}{2}\times\frac{1}{25}+\frac{1}{4}\times\frac{1}{25}+\frac{1}{4}\times\frac{1}{20}$

$=\frac{1}{50}+\frac{1}{100}+\frac{1}{80}=\frac{8+4+5}{400}$

$=\frac{17}{400}=0.0425$

9. Let event A : first red ball;

event B : second red ball,

event C : third red ball

Probability of first red ball, $P(A)=\frac{5}{12}$

remaining balls, red = 4, black = 4 and white = 3

$\therefore$ Probability of second red ball, $P\left(\frac{B}{A}\right)=\frac{4}{11}$

Remaining balls, red = 3, black = 4 and white = 3

$\therefore$ Probability of third red ball, $P\left(\frac{C}{A\cap B}\right)=\frac{3}{10}$

Hence, required probability (all the three balls are red),

$$P(A\cap B\cap C)=P(A)\cdot P\left(\frac{B}{A}\right)\cdot P\left(\frac{C}{A\cap B}\right)$$
$$=\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}$$

$\Rightarrow$ $P(A\cap B\cap C)=\frac{1}{22}$

10. Let event K = card drawn is a king

and event A = card drawn is an ace

Now, $P(K)=\frac{4}{52}$

Also, $P\left(\frac{K}{K}\right)$ is the probability of second king with the condition that one king has already been drawn. Now, there are three kings in $(52-1)=51$ cards.

$\therefore$ $P\left(\frac{K}{K}\right)=\frac{3}{51}$

Lastly, $P\left(\frac{A}{KK}\right)$ is the probability of third drawn card to be an ace, with the condition that two kings have already been drawn. Now, there are four aces in left 50 cards.

$$\therefore \quad P\left(\frac{A}{KK}\right) = \frac{4}{50}$$

By multiplication law of probability,

$$P(KKA) = P(K)\cdot P\left(\frac{K}{K}\right)\cdot P\left(\frac{A}{KK}\right)$$

$$= \frac{4}{52}\cdot\frac{3}{51}\cdot\frac{4}{50} = \frac{2}{5525}$$

11. (i) A and B are independent events

$$\therefore \quad P(A\cap B) = P(A)\cdot P(B)$$

Now, $P(A'\cap B') = 1 - P(A\cup B)$

$$= 1 - [P(A) + P(B) - P(A\cap B)]$$

$$= 1 - P(A) - P(B) + P(A)\cdot P(B)$$

$$[\because P(A\cap B) = P(A)\cdot P(B)]$$

$$= [1 - P(A)][1 - P(B)]$$

$$\Rightarrow \quad P(A'\cap B') = P(A')\cdot P(B')$$

Therefore, A' and B' are also independent events.

Hence proved.

(ii) $$P\left(\frac{B}{A}\right) = \frac{P(B\cap A)}{P(A)} = \frac{P(B)\cdot P(A)}{P(A)}$$

[$\because$ A and B are independent events]

$$= P(B)$$

and $$P\left(\frac{B}{A'}\right) = \frac{P(A'\cap B)}{P(A')} = \frac{P(A')\cdot P(B)}{P(A')} = P(B)$$

Therefore, $P\left(\frac{B}{A}\right) = P(B)$ and $P\left(\frac{B}{A'}\right) = P(B)$.

Hence proved.

12. Let E be the event that threw a pair of dice reports a doublet. Again let S_1 be event that it comes a doublet and S_2 be the event that it does not come a doublet.

Then, $P(S_1) = \frac{6}{36} = \frac{1}{6}$ and $P(S_2) = 1 - \frac{1}{6} = \frac{5}{6}$.

$P(E/S_1)$ = Man speaks a truth $= \frac{7}{10}$

$P(E/S_2)$ = Man does not speak a truth

$$= 1 - \frac{7}{10} = \frac{3}{10}$$

By using Baye's theorem,

$P\left(\frac{S_1}{E}\right)$ = Man speak that it is a doublet,

but it is actually a doublet.

$$= \frac{P(S_1)\times P(E/S_1)}{P(S_1)\times P(E/S_1) + P(S_2)\times P(E/S_2)}$$

$$= \frac{\frac{1}{6}\times\frac{7}{10}}{\frac{1}{6}\times\frac{7}{10} + \frac{5}{6}\times\frac{3}{10}}$$

$$= \frac{\frac{7}{60}}{\frac{7}{60} + \frac{15}{60}} = \frac{7}{22}$$

13. (i) Let E_1 = event that A speaks truth

Then, E'_1 = event that A does not speak truth

E_2 = event that B speaks truth

Then, E'_2 = event that B does not speak truth.

According to the problem, $P(E_1) = \frac{70}{100} = \frac{7}{10}$

$$\therefore \quad P(E'_1) = 1 - P(E_1) = 1 - \frac{7}{10} = \frac{3}{10}$$

$$P(E_2) = \frac{80}{100} = \frac{8}{10}$$

$$\therefore \quad P(E'_2) = 1 - P(E_2) = 1 - \frac{8}{10} = \frac{2}{10}$$

Let E = event that A and B agree with each other.

Now A and B can agree with each other in the following two cases:

(a) A and B, both speak truth this event is denoted by $E_1\cap E_2$.

(b) A and B, both do not speak truth, this event is denoted by $E'_1\cap E'_2$.

Above two events are mutually exclusive.

$$\Rightarrow P(E) = P(E_1\cap E_2) + P(E'_1\cap E'_2)$$

$$= P(E_1)\cdot P(E_2) + P(E'_1)\cdot P(E'_2)$$

$$= \frac{7}{10}\times\frac{8}{10} + \frac{3}{10}\times\frac{2}{10} = \frac{56}{100} + \frac{6}{100} = \frac{62}{100}$$

Therefore, A and B agree with each other in 62% of cases.

(ii) Probability that they oppose each other

= 1 – probability that both agree with each other

$$= 1 - \frac{62}{100} = \frac{38}{100}$$

Therefore, they oppose each other in 38% of cases.

14. Let E = event getting a total of 9

and F = event getting a total of 7.

$$\therefore \quad P(E) = \frac{4}{36} = \frac{1}{9}$$

$$[\because E = \{(4, 5), (5, 4), (3, 6), (6, 3)\}]$$

and $$P(\bar{E}) = 1 - P(E) = 1 - \frac{1}{9} = \frac{8}{9}$$

Also, $P(F) = \frac{6}{36} = \frac{1}{6}$

$$[\because F = \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}]$$

and $P(\bar{F}) = 1 - P(F) = 1 - \frac{1}{6} = \frac{5}{6}$

So, P (B wins the game)

$$= P(\bar{E}F + \overline{EFE}F + \overline{EFEFE}F + \ldots)$$

$$= P(\bar{E})\cdot P(F) + [P(\bar{E})]^2\cdot P(\bar{F})\cdot P(F)$$
$$+ [P(\bar{E})]^3\cdot[P(\bar{F})]^2\cdot[P(F)] + \ldots$$

$$= \frac{P(\bar{E})\cdot P(F)}{1 - P(\bar{E})P(\bar{F})}$$

TR!CK

Sum of infinite terms of G.P. is $S_\infty = \frac{a}{1-r}$

$$= \frac{\frac{8}{9}\times\frac{1}{6}}{1 - \frac{8}{9}\times\frac{5}{6}} = \frac{8}{54}\times\frac{54}{14} = \frac{4}{7}$$

15. Let E_1 be the event that the girl gets 1 or 2.

E_2 be the event that the girl gets 3, 4, 5 or 6 and

A be the event the girl gets exactly a 'tail'.

Then, $P(E_1) = \frac{2}{6} = \frac{1}{3}$ and $P(E_2) = \frac{4}{6} = \frac{2}{3}$

$P\left(\frac{A}{E_1}\right) = P$(getting exactly one tail when a coin is tossed three times) $= \frac{3}{8}$.

$P\left(\frac{A}{E_2}\right) = P$ (getting exactly a tail when a coin is tossed once) $= \frac{1}{2}$.

Now, required probability,

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\cdot P\left(\frac{A}{E_2}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{8}+\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{11}{24}} = \frac{1}{3}\times\frac{24}{11} = \frac{8}{11}.$$

COMMON ERR(!)R

Students forget to define the events and apply Baye's theorem directly and lose marks.

16. Let E_1 be the event that the ghee of type B.

Shop X contains 30 tins of ghee of type A and 40 tins of ghee of type B.

$\therefore$ Probability of ghee of type $B = P(B/X) = \frac{40}{70} = \frac{4}{7}$

Shop Y contains 50 tins of ghee of type A and 60 tins of ghee of type B.

TiP

For Baye's theorem and total probability theorem, events should be well defined.

$\therefore$ Probability of ghee of type $B = P(B/Y) = \frac{60}{110} = \frac{6}{11}$

Also, probability of choosing a shop is $\frac{1}{2}$, as both shop have equal probability of choosing,

$$P(X) = P(Y) = \frac{1}{2}$$

So, required probability,

$$P(Y/B) = \frac{P(Y)\times P(B/Y)}{P(X)\times P(B/X) + P(Y)\times P(B/Y)}$$

$$= \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{11}} = \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\left(\frac{4}{7}+\frac{6}{11}\right)}$$

$$= \frac{6}{11}\times\frac{77}{86} = \frac{21}{43}$$

17. Let, E_1 is event of students which have 100% attendance.

E_2 is event of students which are irregular.

Then, $P(E_1) = \frac{30}{100} = 0.3$

and $P(E_2) = \frac{70}{100} = 0.7$

Let, A = event of students which attendance A grade then

$$P\left(\frac{A}{E_1}\right) = 0.7 \text{ and } P\left(\frac{A}{E_2}\right) = 0.1$$

TiP

For Baye's theorem, events should be well defined.

So, by Baye's theorem,

P (Students has 100% attendance)

$$= P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right) + P(E_2)\cdot P\left(\frac{A}{E_2}\right)}$$

$$= \frac{0.3\times 0.7}{0.3\times 0.7 + 0.7\times 0.1} = \frac{0.3\times 0.7}{0.7(0.3+0.1)} = \frac{0.3}{0.4}$$

$$= \frac{3}{4} = 0.75$$

As per answer, the probability of regular students is more than 50%. So, the regularity is required.

18. Let E_1 = event that selected item is produced from machine A.

TiP

For Baye's theorem and total probability theorem, events should be well defined.

and E_2 = event that selected item is produced from machine B.

Let A = event that produced item is defective

Probability that selected item is produced from machine A

$$P(E_1) = 60\% = 0.6$$

Probability that selected item is produced from machine B

$$P(E_2) = 40\% = 0.4$$

Probability that produced item from machine A is defective

$$P\left(\frac{A}{E_1}\right) = 2\% = 0.02$$

Probability that produced item from machine B is defective

$$P\left(\frac{A}{E_2}\right) = 1\% = 0.01$$

$\therefore$ From Baye's theorem, probability that defective item is produced by machine A when an item is chosen randomly.

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$=\frac{0.6\times0.02}{0.6\times0.02+0.4\times0.01}=\frac{0.012}{0.012+0.004}$$

$$=\frac{0.012}{0.016}=\frac{12}{16}=\frac{3}{4}$$

19. Let E_1 = The event that six comes on the die.

E_2 = The event that six does not comes on the die.

A = The event that man reports it is a six.

$\therefore$ $P(E_1)=\frac{1}{6}$ and $P(E_2)=1-P(E_1)$

$[\because P(E_1)+P(E_2)=1]$

$=1-\frac{1}{6}=\frac{5}{6}$

Probability that the man report that there is a six on the die given that six comes on the die $=P\left(\frac{A}{E_1}\right)$

$=$ Probability that man speaks truth $=\frac{4}{5}$

Probability that the man reports that there is a six on the die given that six does not comes on the die

$=P\left(\frac{A}{E_2}\right)$.

$=$ Probability that man does not speak truth

$=1-\frac{4}{5}=\frac{1}{5}$

By Baye's theorem, we have

$P\left(\frac{E_1}{A}\right)$ = Probability that there is a six given that man reports that there is a six on die.

$$=\frac{P(E_1)\cdot P\left(\frac{A}{E_1}\right)}{p(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}}=\frac{2/15}{\frac{2}{15}+\frac{1}{6}}=\frac{2/15}{\frac{4+5}{30}}$$

$$=\frac{2}{15}\times\frac{30}{9}=\frac{4}{9}$$

COMMON ERR(!)R

Mostly students have a confusion whether to apply total probability theorem or Baye's theorem.

20. Let E_1, E_2 and E_3 be the events that boxes I, II and III are chosen respectively.

Then, $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$

Let A be the event that coin drawn is of gold.

Then, $P(A/E_1)=P$ (a gold coin from box I) $=\frac{2}{2}=1$

$P(A/E_2)=P$ (a gold coin from box II) $=0$

$P(A/E_3)=P$ (a gold coin from box III) $=\frac{1}{2}$

Now, the probability that the other coin in the box is of gold = the probability that gold coin in drawn from the box I $=P(E_1/A)$

$\therefore$ From Baye's theorem,

$P(E_1/A)$

$$=\frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P\left(\frac{A}{E_1}\right)+P(E_2)\cdot P\left(\frac{A}{E_2}\right)+P(E_3)\cdot P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{3}\times1}{\frac{1}{3}\times1+\frac{1}{3}\times0+\frac{1}{3}\times\frac{1}{2}}=\frac{2}{3}$$

COMMON ERR(!)R

Mostly students have a confusion whether to apply total probability theorem or Baye's theorem.

21. Let A, B, C and D be the events that the patient travel by train, bus, scooter and any other means of transport.

$\therefore$ Given that

$P(A)=\frac{3}{10}$ $P(B)=\frac{1}{5}$

$P(C)=\frac{1}{10}$ and $P(D)=\frac{2}{5}$

Let E be the event that he reaches late.

$\therefore$ E/A = event that he reaches late when he travels by train.

E/B = event that he reaches late when he travels by bus.

E/C = event that he reaches late when he travels by scooter.

E/D = event that he reaches late when he travels by any other means.

$\therefore$ $P(E/A)=\frac{1}{4}$ $P(E/B)=\frac{1}{3}$

$P(E/C)=\frac{1}{12}$ and $P(E/D)=0$

Now required probability $=P(A/E)$

$$=\frac{P(A)\cdot P(E/A)}{P(A)\cdot P\left(\frac{E}{A}\right)+P(B)\cdot P\left(\frac{E}{B}\right)+P(C)\cdot P\left(\frac{E}{C}\right)+P(D)\cdot P\left(\frac{E}{D}\right)}$$

$$=\frac{\frac{3}{10}\times\frac{1}{4}}{\frac{3}{10}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{3}+\frac{1}{10}\times\frac{1}{12}+\frac{2}{5}\times0}$$

$$=\frac{\frac{3}{40}}{\frac{3}{40}+\frac{1}{15}+\frac{1}{120}}=\frac{\frac{3}{40}}{\frac{9+8+1}{120}}=\frac{3}{40}\times\frac{120}{18}=\frac{1}{2}$$

COMMON ERR(!)R

Students forget to define the events and apply Baye's theorem directly and lose marks.

22. Given $P(X)=\begin{cases} k, \text{ if } x=0 \\ 2k, \text{ if } x=1 \\ 3k, \text{ if } x=2 \\ 0, \text{ otherwise} \end{cases}$

(i) $\because \sum_{i=1}^{n} P_i = 1$

$\therefore\ k+2k+3k+0=1 \Rightarrow 6k=1$

$\Rightarrow\ k=\frac{1}{6}$

(ii) $P(X<2)=P(X=0)+P(X=1)$

$=k+2k=3\times\frac{1}{6}=\frac{1}{2}$

(iii) $P(X>2)=P(X=3)+P(X=4)+\ldots$

$=0+0+\ldots=0$

23. It is given that the random variable X can take only the values 0, 1, 2, 3.

Given, $P(2)=P(3)=p$

and $P(0)=2P(1)$

Let $P(1)=q$, then $P(0)=2q$

Now, $P(X=0)+P(X=1)+P(X=2)+P(X=3)=1$

TR!CK

Let X be a random variable which can take 'n' values $x_1, x_2, \ldots, x_n$ and $p_1, p_2, \ldots, p_n$ respective probabilities, then

$p_1+p_2+p_3+\ldots+p_n=1.$

$\Rightarrow 2q+q+p+p=1$

$\Rightarrow 3q+2p=1$

$\Rightarrow q=\frac{1-2p}{3}$ ...(1)

Since, $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$

$\Rightarrow p_0x_0^2+p_1x_1^2+p_2x_2^2+p_3x_3^2$

$=2\{p_0x_0+p_1x_1+p_2x_2+p_3x_3\}$

$\Rightarrow 2q\cdot 0+q\cdot(1)^2+P\cdot 2^2+P\cdot 3^2$

$=2\{2q\cdot 0+q\cdot 1+p\cdot 2+p\cdot 3\}$

$\Rightarrow 0+q+p(4+9)=2(0+q+2p+3p)$

$\Rightarrow q+13p=2(q+5p)$

$\Rightarrow q+13p=2q+10p$

$\Rightarrow 3p=q=\frac{1-2p}{3}$ [From eq. (1)]

$\Rightarrow 9p=1-2p$

$\Rightarrow 11p=1 \quad \therefore \quad p=\frac{1}{11}$

24.

TiP

Practice more problems with biased coins and dice.

Here, $P(\text{Head})=\frac{3}{4}$ and $P(\text{Tail})=\frac{1}{4}$

Let X be the number of tails in two cases.

Clearly X can be 0, 1 and 2.

$\therefore$ Probability distribution is given by,

X	0	1	2
$P(X)$	$\frac{3}{4}\times\frac{3}{4}=\frac{9}{16}$	$2\times\frac{3}{4}\times\frac{1}{4}=\frac{6}{16}$	$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$

So, Mean $=\Sigma X P(X)=X_0P_0+X_1P_1+X_2P_2$

$=0\times\frac{9}{16}+1\times\frac{6}{16}+2\times\frac{1}{16}=\frac{8}{16}=\frac{1}{2}$

COMMON ERR(!)R

Mostly students have no idea in finding the probability of head and tail in case of biased coins.

25. Let X denotes the random variable defined by the number of defective items.

$\therefore\ P(X=0)=\frac{4}{6}\times\frac{3}{5}=\frac{2}{5}$

$P(X=1)=2\times\left(\frac{2}{6}\times\frac{4}{5}\right)=\frac{8}{15}$

$P(X=2)=\frac{2}{6}\times\frac{1}{5}=\frac{1}{15}$

So, probability distribution is as follows:

x_i	0	1	2
p_i	2/5	8/15	1/15

Thus, mean $=\Sigma p_ix_i=p_1x_1+p_2x_2+p_3x_3$

$=0\times\frac{2}{5}+1\times\frac{8}{15}+2\times\frac{1}{15}=\frac{10}{15}=\frac{2}{3}$

26. It is given that out of 30 bulbs, 6 are defective.

Therefore, number of non-defective bulbs

$=30-6=24$

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

In a given sample we have to drawn 2 bulbs.

i.e., $X=0, 1, 2$

Here, probability of defective bulb is, $p=\frac{6}{30}=\frac{1}{5}$

and probability of non-defective bulb, $q=\frac{24}{30}=\frac{4}{5}$

Now $P(X=0)=P$ (Two non-defective bulbs)

$=q\times q$

$=\frac{4}{5}\times\frac{4}{5}=\frac{16}{25}$

$P(X=1)=P$

(one defective and one non-defective bulbs)

$=pq+qp$

$=\frac{1}{5}\times\frac{4}{5}+\frac{4}{5}\times\frac{1}{5}$

$=\frac{4}{25}+\frac{4}{25}=\frac{8}{25}$

$P(X=2)=P$ (Two defective bulbs)

$=p\times p=\frac{1}{5}\times\frac{1}{5}=\frac{1}{25}$

The probability distribution of X is:

X	0	1	2
$P(X)$	$\frac{16}{25}$	$\frac{8}{25}$	$\frac{1}{25}$

$\therefore$ Mean $(X) = E(X) = X_1 p_1 + X_2 p_2 + X_3 p_3$

$= 0 \times \frac{16}{25} + 1 \times \frac{8}{25} + 2 \times \frac{1}{25} = \frac{8}{25} + \frac{2}{25}$

$= \frac{10}{25} = \frac{2}{5}$

27. Probability of getting six in single die $= \frac{1}{6}$

Probability of getting non six in single die $= \frac{5}{6}$

Let X represents the numbers of sixes obtained.

When two dice are thrown simultaneously. Therefore X can take the value of 0, 1 or 2.

Now $P(X = 0) = P$ (getting non six on both dice)

$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(X = 1) = P$ (getting six on first die and non six on second die) $+ P$ (getting non six on first die and six on second die)

$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{10}{36}$

$P(X = 2) = P$ (getting six on both dice)

$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$\therefore$ Mean of $X = 0 \times P(X = 0) + 1 \times P(X = 1) + 2 \times P(X = 2)$

$= 0 + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$

$= \frac{12}{36} = \frac{1}{3}$

28. Total number of possible outcomes $= {}^5P_2$

TiP

Practice more problems on elementary probability involving permutations and combinations.

$= \frac{5!}{3!} = 5 \times 4 = 20$

Here, X denotes the larger of two numbers obtained.

$\therefore$ X can take values 2, 3, 4 and 5.

Now, $P(X = 2) = P$ {getting (1, 2) or (2, 1)} $= \frac{2}{20} = \frac{1}{10}$

$P(X = 3) = P$ {getting (1, 3) or (3, 1) or (2, 3) or (3, 2)}

$= \frac{4}{20} = \frac{1}{5}$

$P(X = 4) = P$ {getting (1, 4) or (4, 1) or (2, 4) or (4, 2) or (3, 4) or (4, 3)} $= \frac{6}{20} = \frac{3}{10}$

and $P(X = 5) = P$ {getting (1, 5) or (5, 1) or (2, 5) or (5, 2) or (3, 5) or (5, 3) or (4, 5) or (5, 4)} $= \frac{8}{20} = \frac{2}{5}$

TR!CK

Let X be a random variable taking values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively. Then mean of a random variable X is $\sum_{i=1}^{n} x_i p_i$.

Thus, the probability distribution of X is:

X	2	3	4	5
$P(X)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

Now, Mean of $X = E(X) = \Sigma X \cdot P(X)$

$= X_2P_2 + X_3P_3 + X_4P_4 + X_5P_5$

$= 2 \cdot \frac{1}{10} + 3 \cdot \frac{1}{5} + 4 \cdot \frac{3}{10} + 5 \cdot \frac{2}{5}$

$= \frac{1}{10}(2 + 6 + 12 + 20) = \frac{40}{10} = 4.$

29. Here, $S = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\} \Rightarrow n(S) = 12$

Let random variable X denotes the sum of the numbers on two cards drawn. So, the random variables X may have values 3, 4, 5, 6 and 7.

TiP

Let X be a random variable which can take n values $x_1, x_2, \ldots, x_n$. Let $p_1, p_2, \ldots, p_n$ be the respective probabilities, then $p_1 + p_2 + p_3 + \ldots + p_n = 1$ where, $p_i > 0$, $i = 1, 2, \ldots, n$.

At $X = 3$, $P(X) = \frac{2}{12} = \frac{1}{6}$ At $X = 4$, $P(X) = \frac{2}{12} = \frac{1}{6}$

At $X = 5$, $P(X) = \frac{4}{12} = \frac{1}{3}$ At $X = 6$, $P(X) = \frac{2}{12} = \frac{1}{6}$

At $X = 7$, $P(X) = \frac{2}{12} = \frac{1}{6}$

TR!CK

Let X be a random variable taking values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively. Then mean of a random variable X is $\sum_{i=1}^{n} x_i p_i$.

Therefore, the required probability distribution is as follows:

X	3	4	5	6	7
$P(X)$	1/6	1/6	1/3	1/6	1/6

$\therefore$ Mean, $E(X) = \Sigma X P(X)$

$= X_3P_3 + X_4P_4 + X_5P_5 + X_6P_6 + X_7P_7$

$= 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{3} + 6 \times \frac{1}{6} + 7 \times \frac{1}{6}$

$= \frac{3}{6} + \frac{4}{6} + \frac{10}{6} + \frac{6}{6} + \frac{7}{6} = \frac{30}{6} = 5$

Long Answer Type Questions

1. Let E_1 = event, student knows the answer

TiP

For Baye's theorem, events should be well defined.

and E_2 = event, student guesses the answer, then

$P(E_1) = \frac{3}{5}$, $P(E_2) = \frac{2}{5}$

Let A = events, student gives the right answer.

If student knows the answer $\Rightarrow$ answer is correct.

$\therefore$ Probability that if answer knows, answer is correct

$$= P\left(\frac{A}{E_1}\right) = 1$$

Probability that if answer he gusses, answer is correct $= P\left(\frac{A}{E_2}\right) = \frac{1}{3}$

By Baye's theorem, probability that student knows the answer of question, if it is known that he gives right answer:

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{3}{5}\times 1}{\frac{3}{5}\times 1 + \frac{2}{5}\times\frac{1}{3}} = \frac{\frac{3}{5}}{\frac{3}{5}+\frac{2}{15}} = \frac{3}{5}\times\frac{15}{11} = \frac{9}{11}$$

2. Given, $P(A\cap B) = P(A)\cdot P(B) = \frac{1}{6}$ ...(1)

and $P(A'\cap B') = \frac{1}{3}$

but $P(A'\cap B') = 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)]$

$\Rightarrow \frac{1}{3} = 1 - P(A) - P(B) + P(A\cap B)$

$\Rightarrow P(A) + P(B) = 1 - \frac{1}{3} + \frac{1}{6} = 1 - \frac{1}{6} = \frac{5}{6}$

$\Rightarrow P(B) = \frac{5}{6} - P(A)$

then from eq. (1),

$$P(A)\left[\frac{5}{6} - P(A)\right] = \frac{1}{6}$$

$\Rightarrow \frac{5}{6}P(A) - [P(A)]^2 = \frac{1}{6}$

$\Rightarrow 5P(A) - 6[P(A)]^2 = 1$

$\Rightarrow 6[P(A)]^2 - 5P(A) + 1 = 0$

$\Rightarrow 6[P(A)]^2 - 3P(A) - 2P(A) + 1 = 0$

$\Rightarrow (2P(A) - 1)(3P(A) - 1) = 0$

$\Rightarrow P(A) = \frac{1}{2}$ or $P(A) = \frac{1}{3}$

Now, $P(B) = \frac{1}{6P(A)}$ [From eq. (1)]

when $P(A) = \frac{1}{2}$, then $P(B) = \frac{1}{3}$

when $P(A) = \frac{1}{3}$, then $P(B) = \frac{1}{2}$

Therefore, required probabilities are $\frac{1}{2}$ and $\frac{1}{3}$

3. Let A : Event that item produced by operator A.

B : Event that item produced by operator B.

C : Event that item produced by operator C.

D : Event that item produced as defective.

Now, $P(A) =$ Probability of item is produced by operator A

$$= 50\% = \frac{50}{100} = 0.5$$

$P(B) =$ Probability of item is produced by operator B

$$= 30\% = \frac{30}{100} = 0.3$$

$P(C) =$ Probability of item is produced by operator C

$$= 20\% = \frac{20}{100} = 0.2$$

$P(D/A) =$ Probability of a defective item produced by operator A

$$= 1\% = \frac{1}{100} = 0.01$$

$P(D/B) =$ Probability of a defective item produced by operator B

$$= 5\% = \frac{5}{100} = 0.05$$

$P(D/C) =$ Probability of a defective item produced by operator C

$$= 7\% = \frac{7}{100} = 0.07$$

$\therefore$ Probability that item is produced by operator A if it is defective *i.e.*, $P(A/D)$

$$= \frac{P(A)\cdot P(D/A)}{P(A)\cdot P(D/A) + P(B)\cdot P(D/B) + P(C)P(D/C)}$$

$$= \frac{0.5\times 0.01}{0.5\times 0.01 + 0.3\times 0.05 + 0.2\times 0.07}$$

$$= \frac{0.005}{(0.005 + 0.015 + 0.014)} = \frac{0.005}{0.034} = \frac{5}{34}$$

COMMON ERR(!)R

Mostly students have a confusion whether to apply total probability theorem or Baye's theorem.

4. Let E_1 : Event that urn has 2 white balls.

E_2 : Event that urn has 3 white balls.

E_3 : Event that urn has 4 white balls.

A : Event that 2 balls drawn are white.

Now, $P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{3}$.

and $P(A/E_1) = \frac{^2C_2}{^4C_2} = 1\times\frac{2}{4\times 3} = \frac{1}{6}$,

$P(A/E_2) = \frac{^3C_2}{^4C_2} = \frac{3\times 2}{2\times 1}\times\frac{2\times 1}{4\times 3} = \frac{1}{2}$,

$P(A/E_3) = \frac{^4C_2}{^4C_2} = 1$.

$\therefore$ Required probability,

$P(E_3/A)$

$$= \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times\frac{1}{6} + \frac{1}{3}\times\frac{1}{2} + \frac{1}{3}\times 1} = \frac{1}{\frac{1}{6}+\frac{1}{2}+1}$$

$$= \frac{6}{1+3+6} = \frac{6}{10} = 0.6$$

COMMON ERR(!)R

Students forget to define the event and apply Baye's theorem directly and lose marks.

5. The probability of drawing a ticket with a prize of ₹ 8 is, $P(₹\,8) = \frac{2}{10} = \frac{1}{5}$

The probability of drawing a ticket with a prize of ₹ 4 is, $P(₹\,4) = \frac{5}{10} = \frac{1}{2}$

The probability of drawing a ticket with a prize of ₹ 2 is, $P(₹\,2) = \frac{3}{10}$

∴ Mean value of the prize

$$= P(₹\,8)\times 8 + P(₹\,4)\times 4 + P(₹\,2)\times 2$$
$$= \frac{1}{5}\times 8 + \frac{1}{2}\times 4 + \frac{3}{10}\times 2$$
$$= \frac{8}{5} + \frac{4}{2} + \frac{6}{10} = \frac{16+20+6}{10}$$
$$= \frac{42}{10} = 4.2$$

6. The sample space of the experiment consists of 36 elementary events in the form of ordered pairs (x, y) where $x_i = 1, 2, 3, 4, 5, 6$ and $y_i = 1, 2, 3, 4, 5, 6$.

The random variable X, *i.e.*, the sum of the numbers on the two dice takes the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 or 12.

TIP
At random means without replacement.

Now, $P(X=2) = P(\{(1, 1)\}) = \frac{1}{36}$

$P(X=3) = P(\{(1, 2), (2, 1)\}) = \frac{2}{36}$

$P(X=4) = P(\{(1, 3), (2, 2), (3, 1)\}) = \frac{3}{36}$

$P(X=5) = P(\{(1, 4), (2, 3), (3, 2), (4, 1)\}) = \frac{4}{36}$

$P(X=6) = P(\{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}) = \frac{5}{36}$

$P(X=7) = P(\{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}) = \frac{6}{36}$

$P(X=8) = P(\{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}) = \frac{5}{36}$

$P(X=9) = P(\{(3, 6), (4, 5), (5, 4), (6, 3)\}) = \frac{4}{36}$

$P(X=10) = P(\{(4, 6), (5, 5), (6, 4)\}) = \frac{3}{36}$

$P(X=11) = P(\{(5, 6), (6, 5)\}) = \frac{2}{36}$

$P(X=12) = P(\{(6, 6)\}) = \frac{1}{36}$

The probability distribution of X is:

X or x_i	2	3	4	5	6	7	8	9	10	11	12
$P(X)$ or p_i	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\mu = E(X) = \sum_{i=1}^{n} x_i p_i = 2\times\frac{1}{36} + 3\times\frac{2}{36} + 4\times\frac{3}{36} + 5\times\frac{4}{36} + 6\times\frac{5}{36} + 7\times\frac{6}{36} + 8\times\frac{5}{36} + 9\times\frac{4}{36} + 10\times\frac{3}{36} + 11\times\frac{2}{36} + 12\times\frac{1}{36}$$
$$= (2+6+12+20+30+42+40+36+30+22+12)\times\frac{1}{36}$$
$$= \frac{252}{36} = 7$$

Thus, the mean of the sum of numbers that appear on throwing two fair dice is 7.

7. Let X be the number of kings obtained.

We can get 0, 1 or 2 kings. So, value of X is 0, 1 or 2.

Total number of ways to draw 2 cards out of 52

i.e., Total ways $= {}^{52}C_2 = 1326$

$P(X=0)$ *i.e.*, Probability of getting 0 king.

TIP
Practice more problems on elementary probability involving combinations.

Number of ways to get 0 king

= Number of ways to select 2 cards out of non-king cards

= Number of ways to select 2 cards out of $(52-4)$ = 48 cards $= {}^{48}C_2 = 1128$

$$\therefore\ P(X=0) = \frac{\text{Number of ways to get 0 king}}{\text{Total number of ways}} = \frac{1128}{1326}$$

$P(X=1)$ *i.e.*, Probability of getting 1 king.

Number of ways to get 1 king

= Number of ways to select 1 king out of 4 king cards × Number of ways to select 1 card from 48 non-king cards

$= {}^{4}C_1 \times {}^{48}C_1 = 4\times 48 = 192$

$$\therefore\ P(X=1) = \frac{\text{Number of ways to get 1 king}}{\text{Total number of ways}} = \frac{192}{1326}$$

$P(X=2)$ *i.e.*, Probability of getting 2 kings.

Number of ways to get 2 kings

= Number of ways of selecting 2 kings out of 4 king cards

$= {}^{4}C_2 = 6$

$$\therefore\ P(X=2) = \frac{\text{Number of ways to get 2 kings}}{\text{Total number of ways}} = \frac{6}{1326}$$

Mean $(\mu) = E(X) = \sum_{i=1}^{n} X_i P_i$

$$= X_1P_1 + X_2P_2 + X_3P_3$$
$$= 0\times\frac{1128}{1326} + 1\times\frac{192}{1326} + 2\times\frac{6}{1326}$$
$$= 0 + \frac{192+12}{1326} = \frac{204}{1326} = \frac{34}{221}$$

TR!CK

Let X be a random variable taking values $x_1, x_2, \ldots, x_n$ with probabilities $p_1, p_2, \ldots, p_n$ respectively. Then mean of a random variable X is $\sum_{i=1}^{n} x_i p_i$.

8. Given, X denotes the smallest of the three numbers obtained.

TiP

Revision must be done on concepts of mean of probability distribution.

$\therefore$ X takes values 1, 2, 3 and 4.

$$P(X=1) = \frac{^5C_2}{^6C_3} = \frac{5\times4}{2}\times\frac{3\times2\times1}{6\times5\times4} = \frac{1}{2}$$

$$P(X=2) = \frac{^4C_2}{^6C_3} = \frac{4\times3}{2}\times\frac{3\times2\times1}{6\times5\times4} = \frac{3}{10}$$

$$P(X=3) = \frac{^3C_2}{^6C_3} = \frac{3\times2}{2\times1}\times\frac{3\times2\times1}{6\times5\times4} = \frac{3}{20}$$

$$P(X=4) = \frac{^2C_2}{^6C_3} = 1\times\frac{3\times2\times1}{6\times5\times4} = \frac{1}{20}$$

$\therefore$ Probability distribution is:

X	1	2	3	4
$P(X)$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$

So, Mean $= \Sigma XP(x)$

$$= X_1P_1 + X_2P_2 + X_3P_3 + X_4P_4$$

$$= 1\times\frac{1}{2} + 2\times\frac{3}{10} + 3\times\frac{3}{20} + 4\times\frac{1}{20}$$

$$= \frac{1}{20}\times(10+12+9+4)$$

$$= \frac{35}{20} = 1.75$$

Chapter Test

Multiple Choice Questions

Q 1. If two cards are drawn from a well shuffled deck of 52 playing cards with replacement, then the probability that both cards are queens, is:

a. $\frac{1}{13}\cdot\frac{1}{13}$ b. $\frac{1}{13}+\frac{1}{13}$

c. $\frac{1}{13}\cdot\frac{1}{17}$ d. $\frac{1}{13}\cdot\frac{4}{51}$

Q 2. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A/B) = \frac{1}{4}$, then $P(A' \cap B')$ equals to:

a. $\frac{1}{12}$ b. $\frac{3}{4}$ c. $\frac{1}{4}$ d. $\frac{3}{16}$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Let E_1 and E_2 be any two events associated with an experiment.

Assertion (A): $P(E_1) + P(E_2) \leq 1$

Reason (R): $P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$

Q 4. Assertion (A): Bag I contains 3 red and 4 black balls while another bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Then, the probability that it was drawn from bag II, is $\frac{35}{68}$.

Reason (R): Given, three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, then the probability that the other coin in the box is also of gold, is $\frac{1}{2}$.

Case Study Based Questions

Q 5. Case Study 1

A coach is training 3 players. He observes that the Player A can hit a target 4 times in 5 shots, Player B can hit 3 times in 4 shots and the Player C can hit 2 times in 3 shots.

In an event, a coach gives an opportunity to all three persons to hit the target.

Based on the given information, solve the following questions:

(i) Find the probability that A, B and C all will hit.

(ii) What is the probability that B, C will hit and A will lose?

(iii) What is the probability that any two of A, B and C will hit?

Or

What is the probability that none of them will hit the target?

Q 6. Case Study 2

By examine the test, the probability that a person is diagnosed with CORONA when he is actually suffering from it, is 0.99. The probability that the doctor incorrectly diagnosed a person to be having CORONA, on the basis of test reports, is 0.001. In a certain city, 1 in 1000 persons suffers from CORONA. A person is selected at random and is diagnosed to have CORONA.

Based on the above information, solve the following questions:

(i) What is the P(CORONA is diagnosed, when the person actually has CORONA)?

(ii) What is the P(CORONA is diagnosed, when the person has not CORONA)?

(iii) What is P(CORONA is diagnosed)?

Or

What is the P(Person has CORONA given CORONA is diagnosed)?

Very Short Answer Type Questions

Q 7. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

Q 8. The probability of A winning the race is $\frac{1}{3}$ and that of B winning the race is $\frac{1}{4}$. What is the probability that both will never win this race?

Short Answer Type-I Questions

Q 9. Given that E and F are events such that $P(E) = 0.8$, $P(F) = 0.7$, $P(E \cap F) = 0.6$. Find $P(\bar{E}/\bar{F})$.

Q 10. A question paper of mathematics is given to three students A, B, C to solve, for which the possibilities of solving are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, then find the probability of solving the question.

Short Answer Type-II Questions

Q 11. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	a	$4a$	$3a$	$7a$	$8a$	$10a$	$6a$	$9a$

Find $P(X < 3)$, $P(X \geq 4)$, $P(0 < X < 5)$ respectively.

Q 12. A random variable X has the following distribution:

X	1	2	3	4	5	6	7	8
$P(X)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, find $P(E \cup F)$.

Long Answer Type Questions

Q 13. Three persons A, B and C apply for a job of Manager in a Private company. Chance of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probability that A, B and C can introduce changes to improve profits of company are 0.8, 0.5 and 0.3 respectively, if the changes does not take place, find the probability that it is due to the appointment of C.

Q 14. A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem respectively, are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{2}{3}$. What is the probability that:

(i) the problem will be solved?

(ii) exactly one of them solves the problem?

(iii) atleast three of them solves the problem?

(iv) atmost one of them solves the problem?